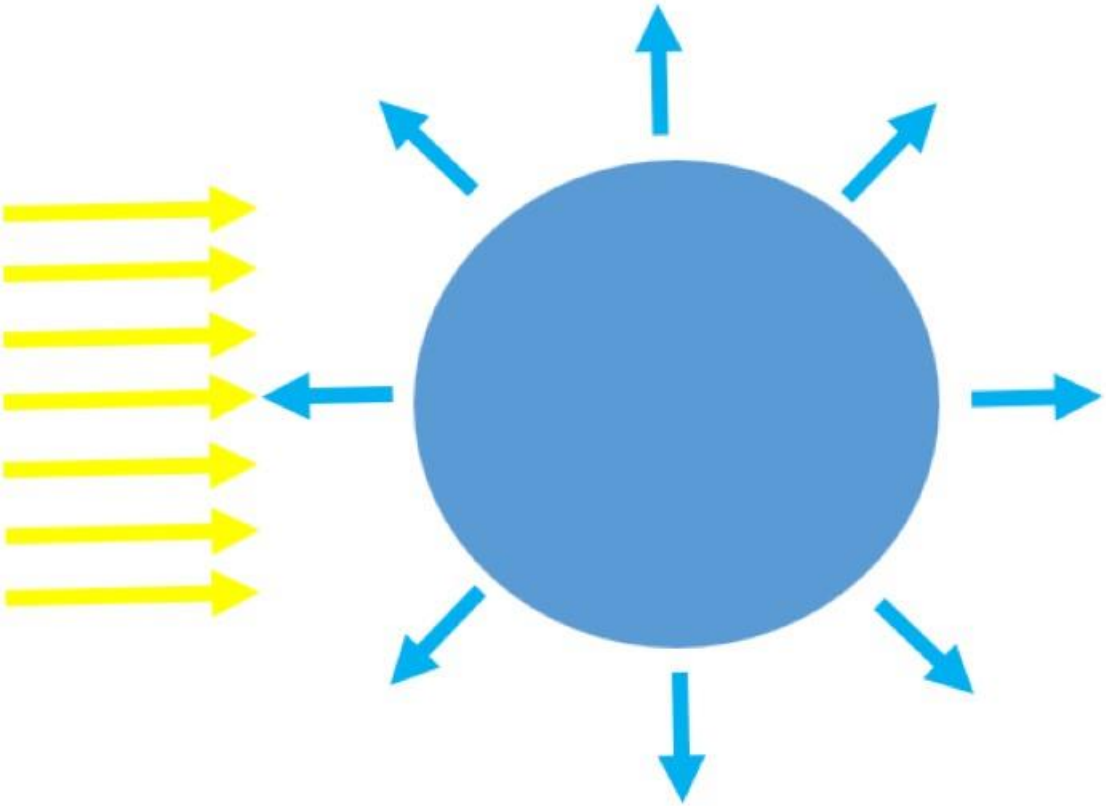
Radiation model for increase in Earth's temperature

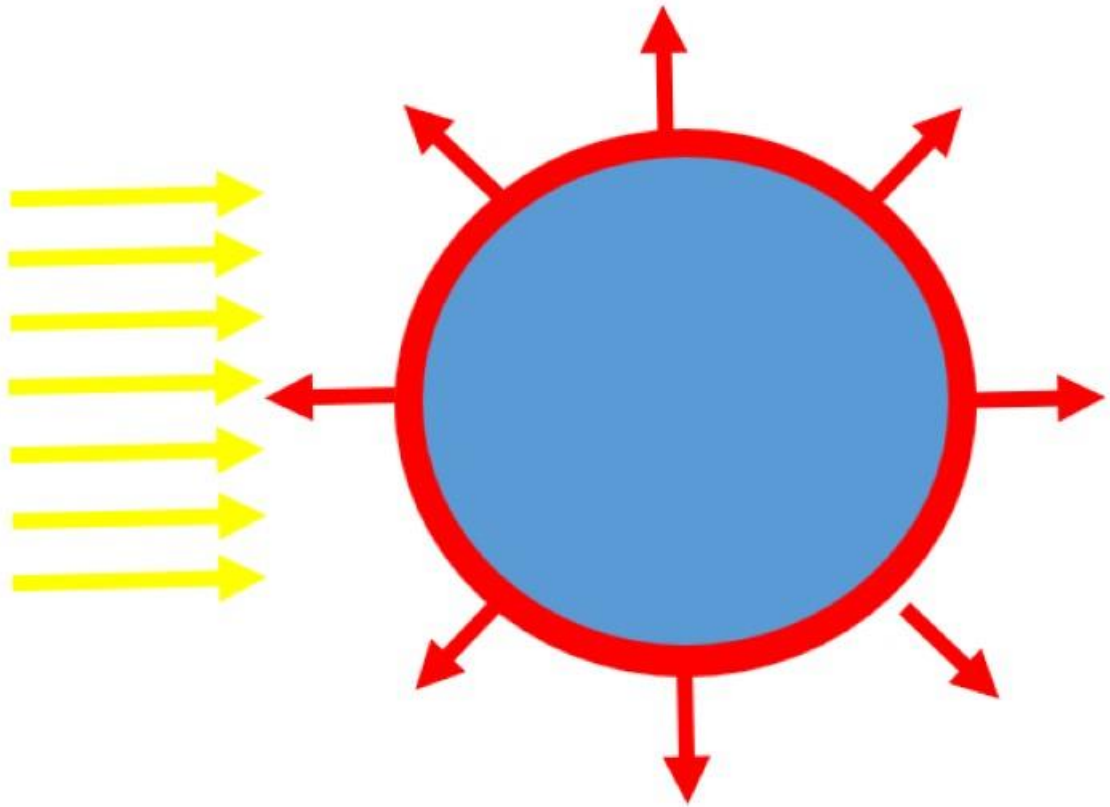
A physical proof that increasing CO_2 in the atmosphere causes surface warming

Bare earth



$$F_{solar} = F_{bare\ earth}$$

Earth with atmosphere



$$F_{solar} = F_{TOA}$$

TOA is “top of atmosphere”

The greenhouse effect of an atmosphere does not change the cosmic radiation equilibrium and therefore

$$F_{TOA} = F_{bare\ earth}$$

Both fluxes are related to the temperature by the Stefan-Boltzmann law.

$$F = \sigma T^4$$

At equilibrium the temperature at the top of the atmosphere is equal to that of the bare earth in this simple model. This is a constraint on the system that can be used to calculate the surface temperature in an atmosphere of known transmittance.

Our simplifying assumption is that the transmitted intensity at the TOA is the surface flux from the in the presence of an absorbing atmosphere, F_{atm}, multiplied by the transmittance, τ_{atm}.

$$F_{TOA} = F_{atm}\tau_{atm}$$

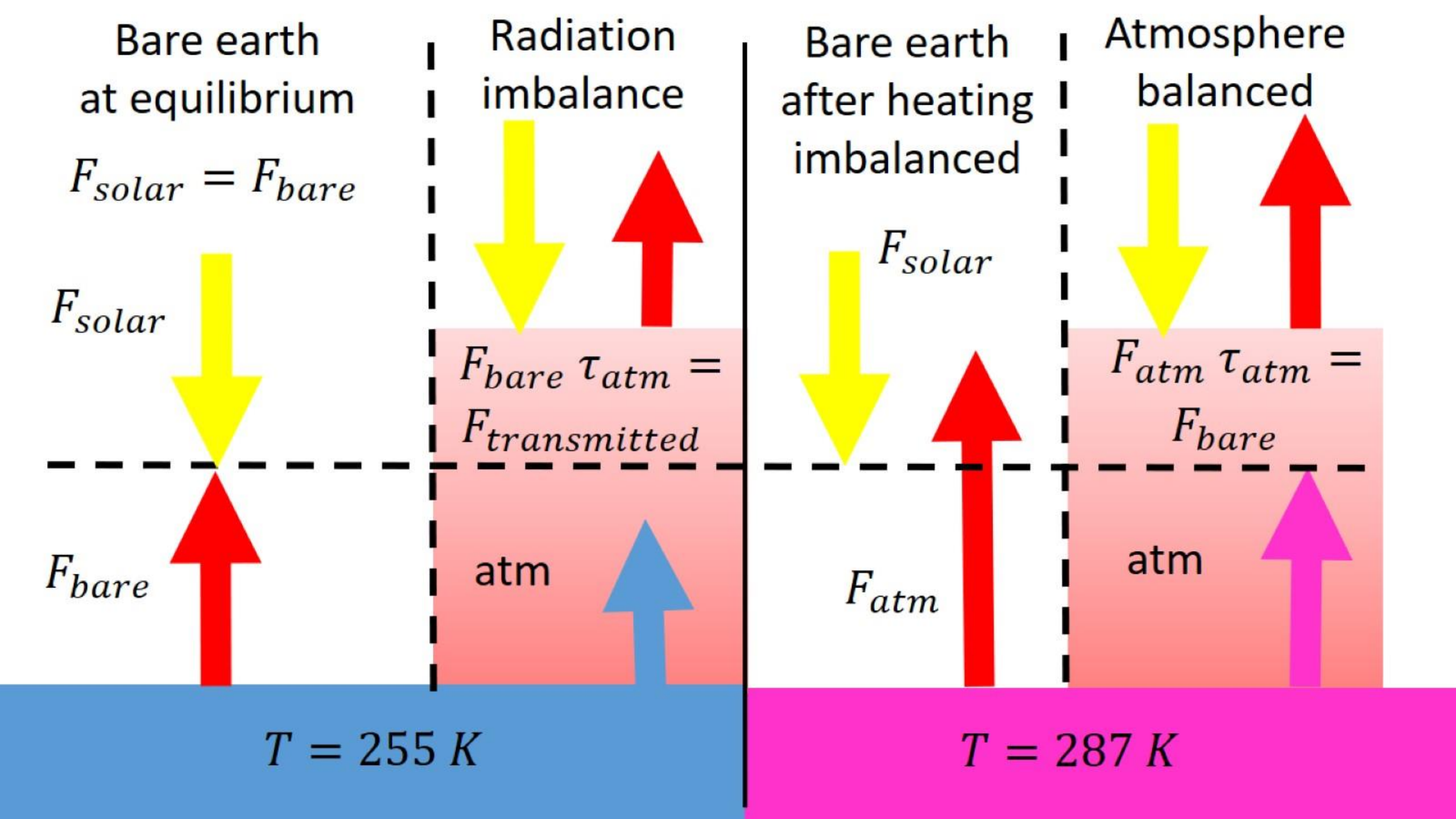
Since F_{TOA} = $F_{bare\ earth}$ we have

$$F_{atm} = \frac{F_{bare\ earth}}{\tau_{atm}}$$

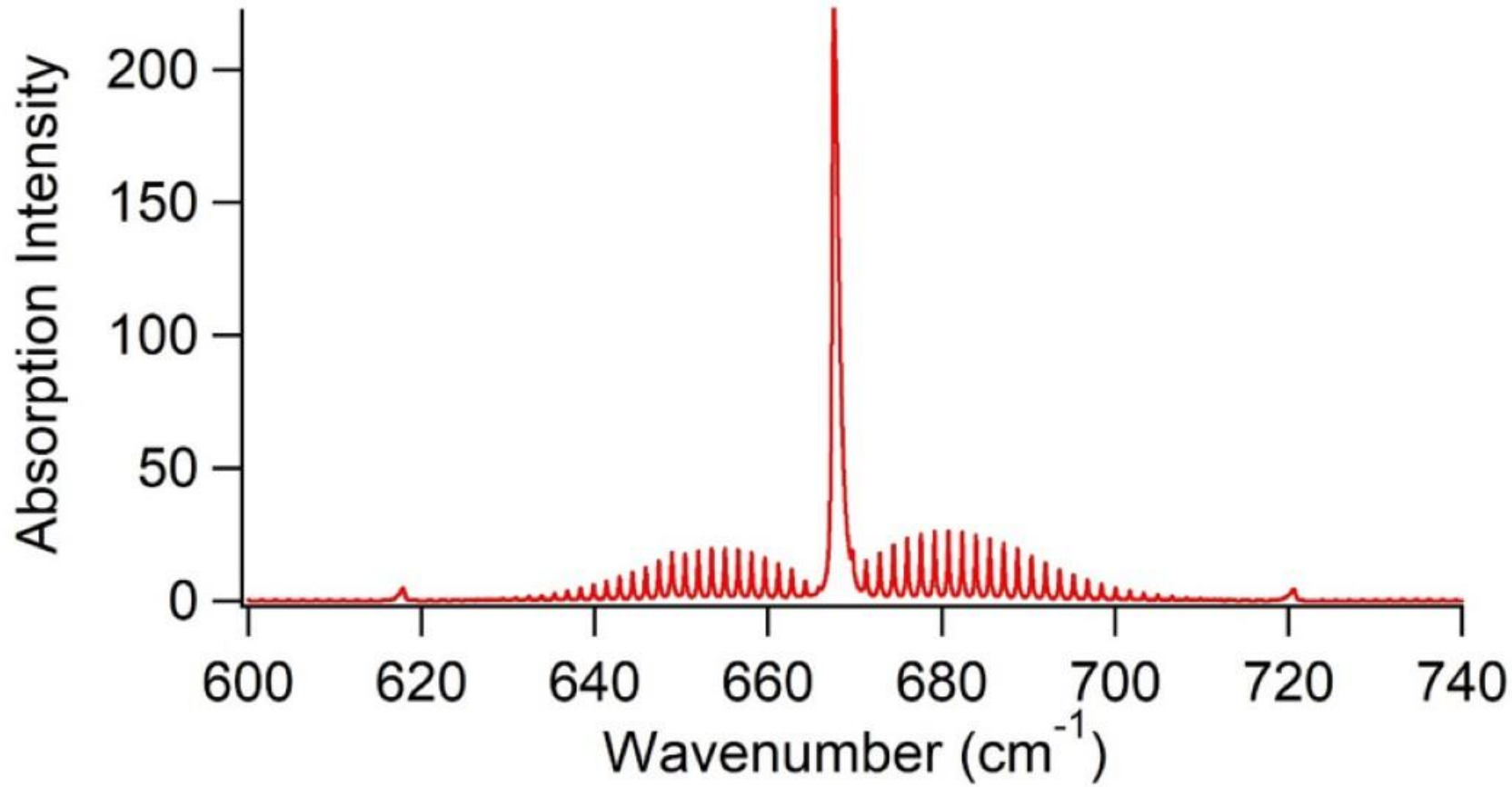
Therefore,

$$T_{atm} = \frac{T_{bare\ earth}}{\sqrt[4]{\tau_{atm}}}$$

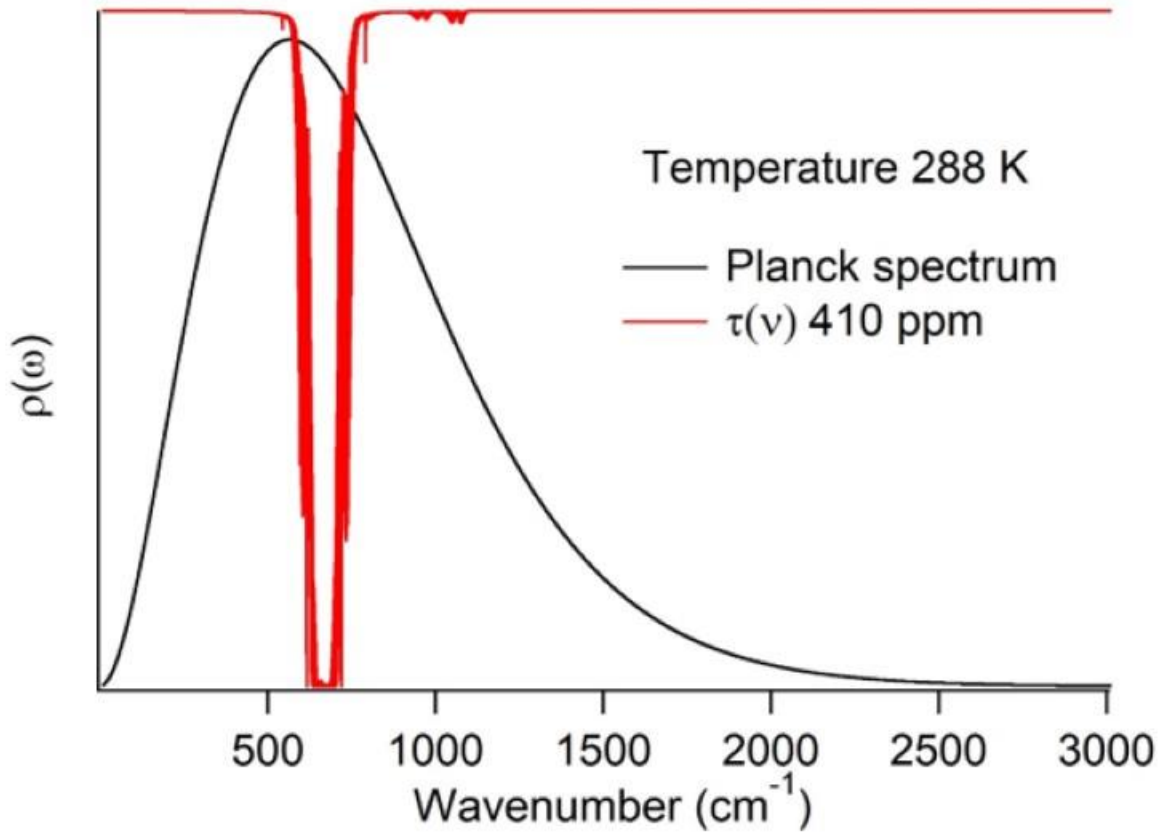
According to this calculation, when $T_{bare\ earth} = 255$ K. The equilibrated surface temperature of the earth is $T_{atm} = 287$ K. We assume an atmosphere with P_{CO_2} = 420 ppm. Use the HITRAN database and Planck equation to determined the transmittance, where we also include water as an average value, $\tau_{atm} = \tau_{CO_2}\ \tau_{H_2O}$. The next pages Illustrate these concepts.

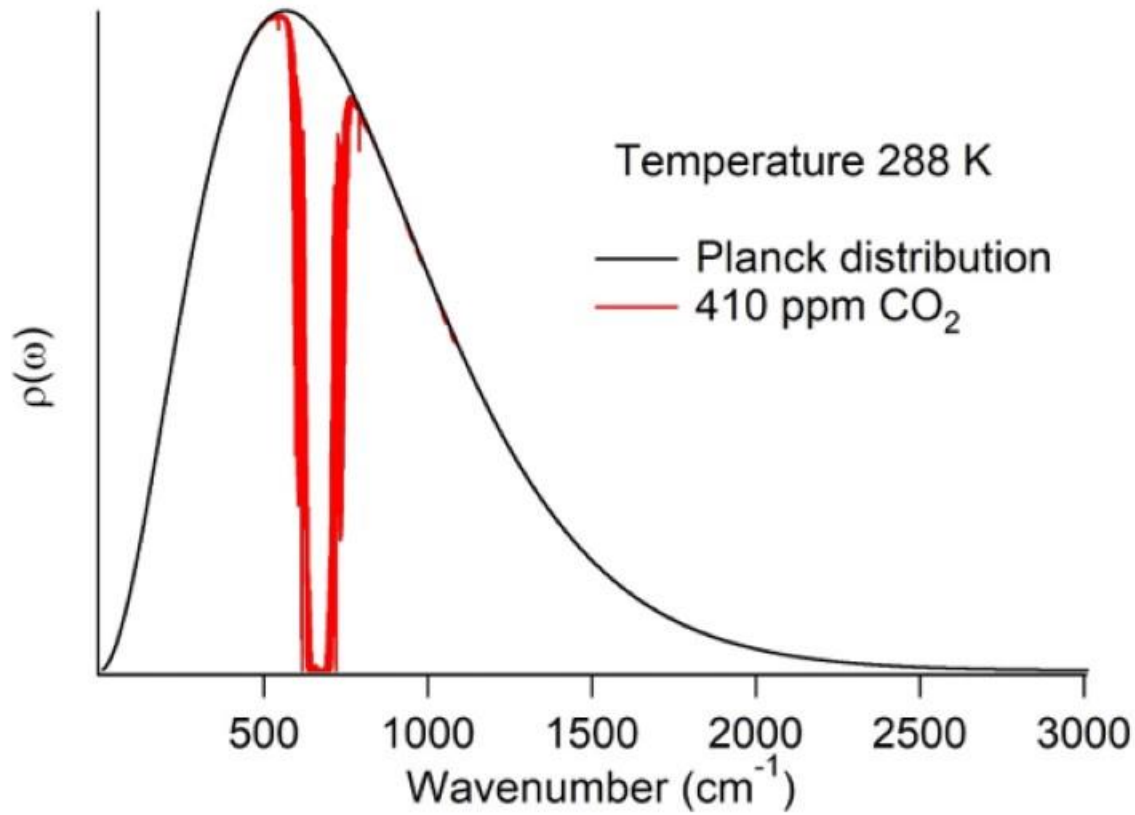

Bare earth
at equilibrium
$F_{solar} = F_{bare}$
F_{solar}
F_{bare}
$T = 255\ K$
Radiation
imbalance
$F_{bare}\ \tau_{atm} =$
$F_{transmitted}$
atm
Bare earth
after heating
imbalanced
F_{solar}
F_{atm}
Atmosphere
balanced
$F_{atm}\ \tau_{atm} =$
F_{bare}
atm
$T = 287\ K$

Using the HITRAN intensity data for natural abundance CO_2 we can Calculate an absorption spectrum using a Lorentzian line shape with a width of $\gamma = 0.13\ cm^{-1}$

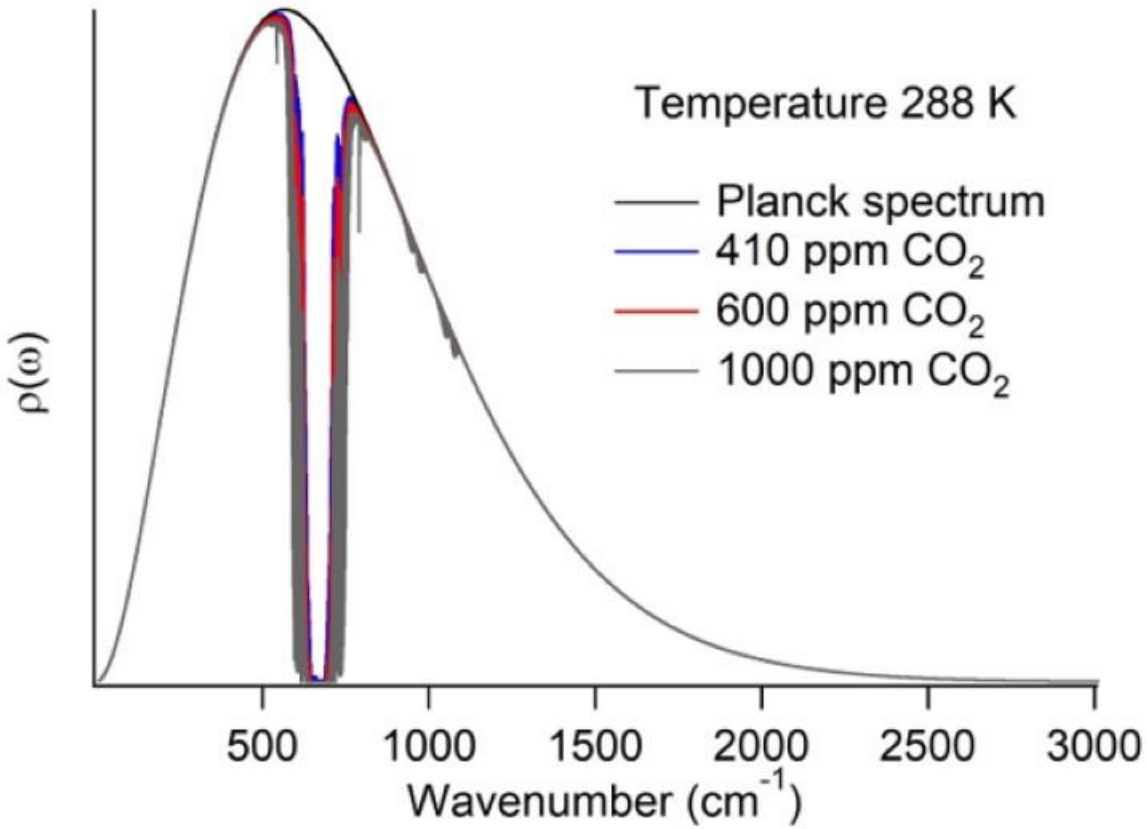


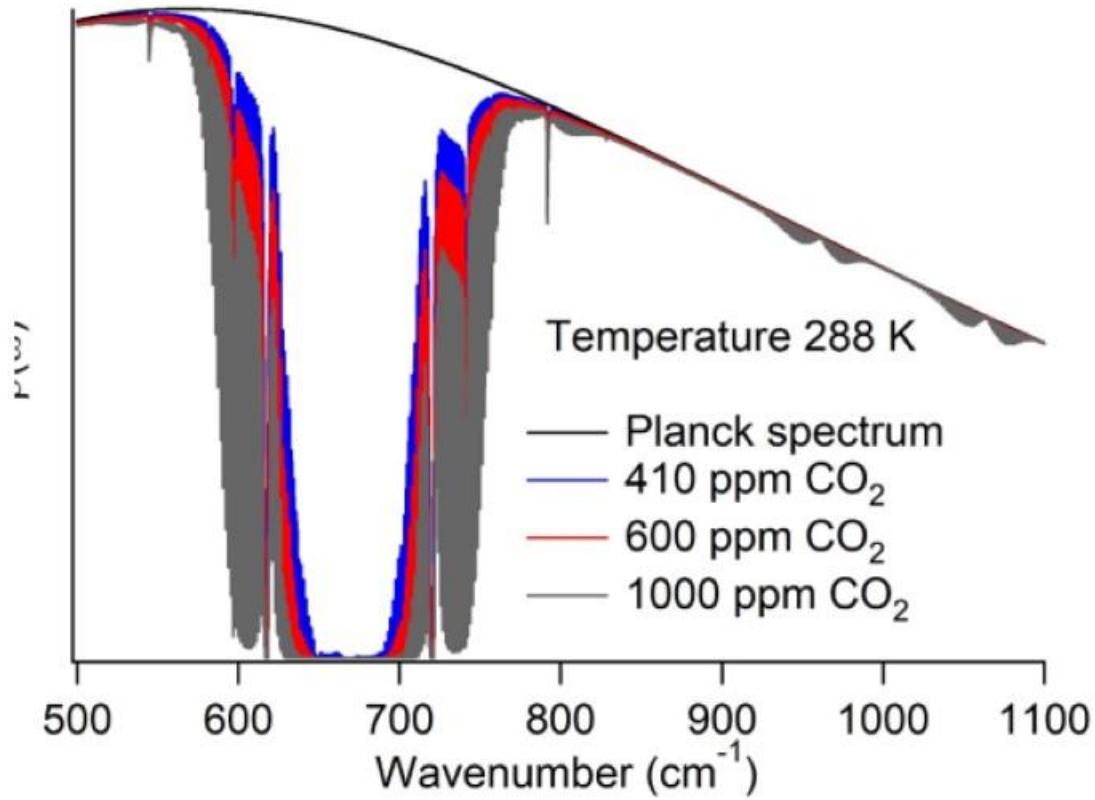
The HITRAN intensity data is then applied to a calculation of transmittance through the atmosphere. The transmittance is 1 except where CO_2 bending mode bands absorb. The overlay is shown with the earth's thermal radiation spectrum. The product is shown on the right.



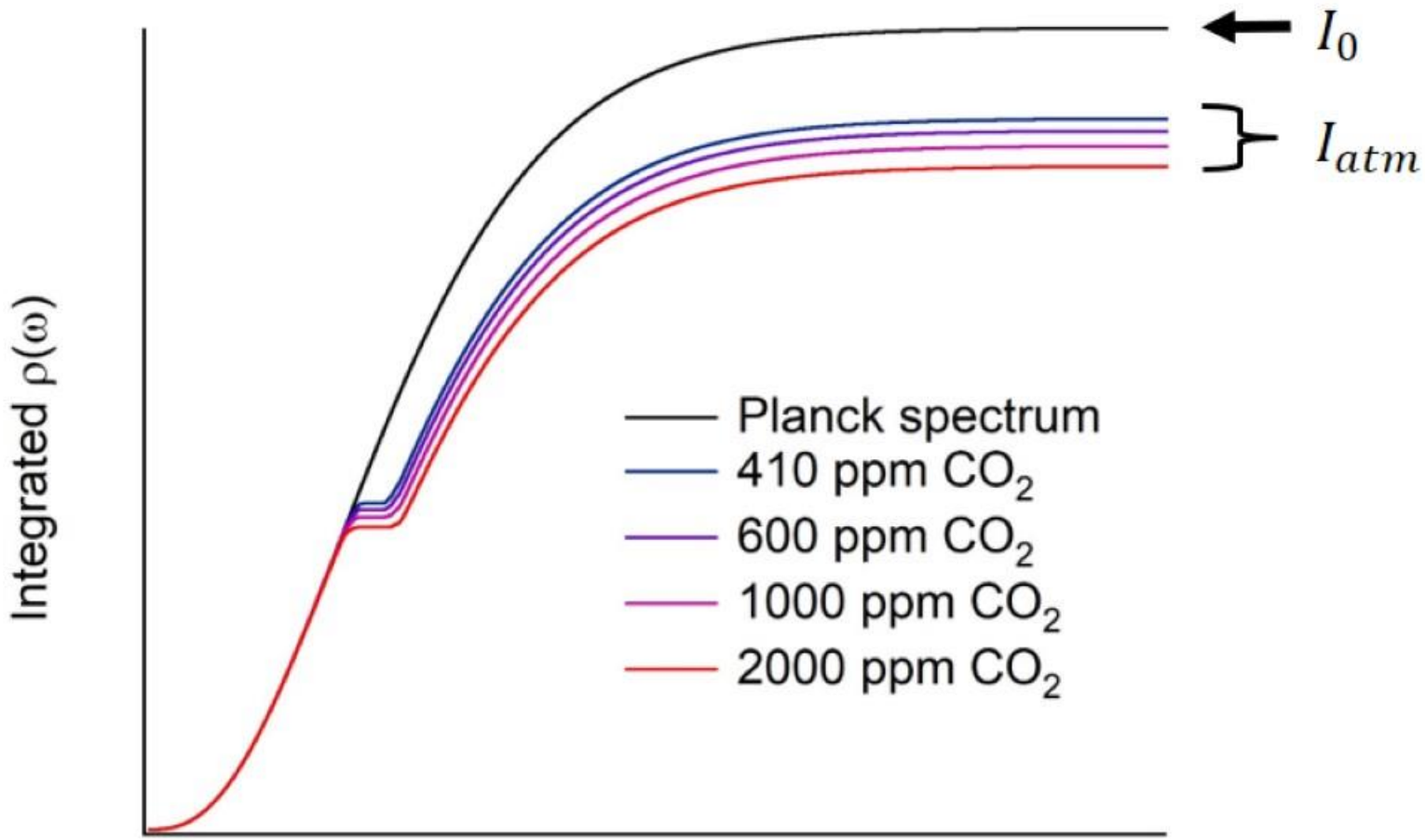


The comparison on this slide shows P_{CO_2} of 410, 600, and 1000 ppm. An expansion of the region around the peak of the Planck spectrum is shown for emphasis.





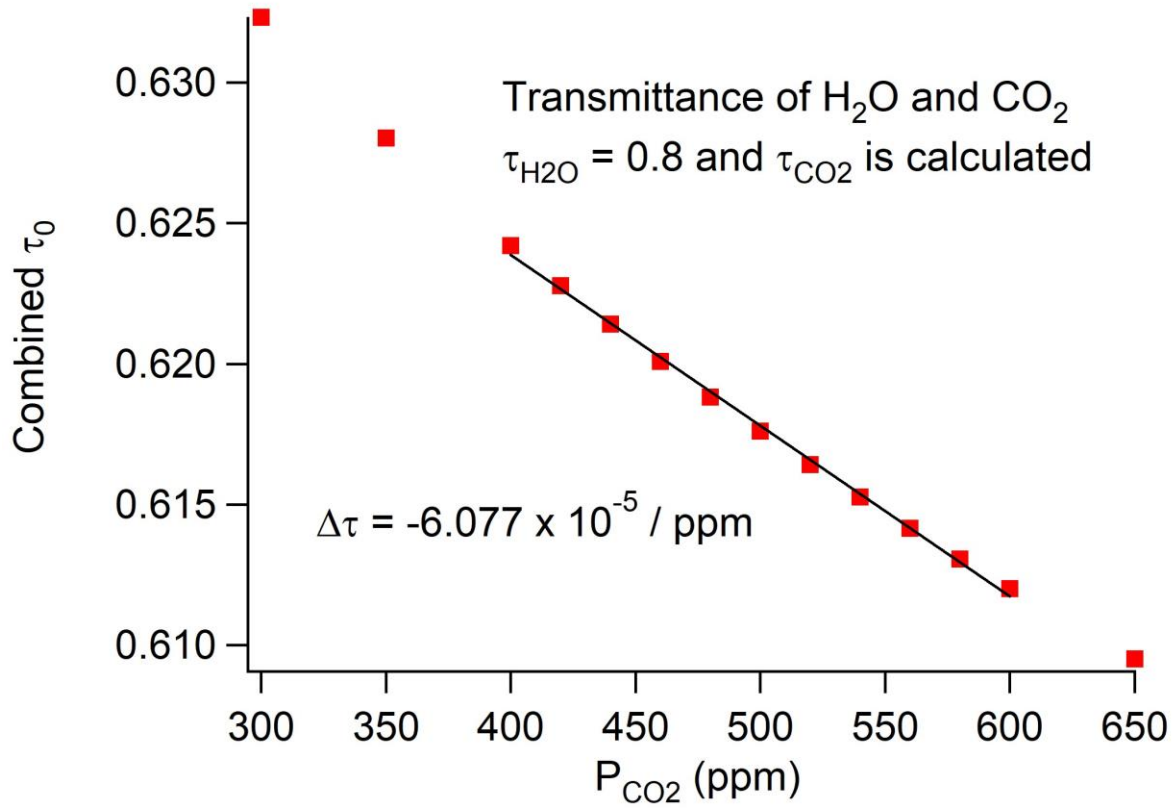
The transmittance of radiation at all frequencies is determined by taking the integral of the Planck function with each of the added transmission regions for CO_2.



Calculated transmittance
At various values of CO_2 ppm

$$\tau_{atm} = \frac{I_{atm}}{I_0}$$

The transmittance calculated for CO_2 only is shown below.



We take water into account using the approximation that

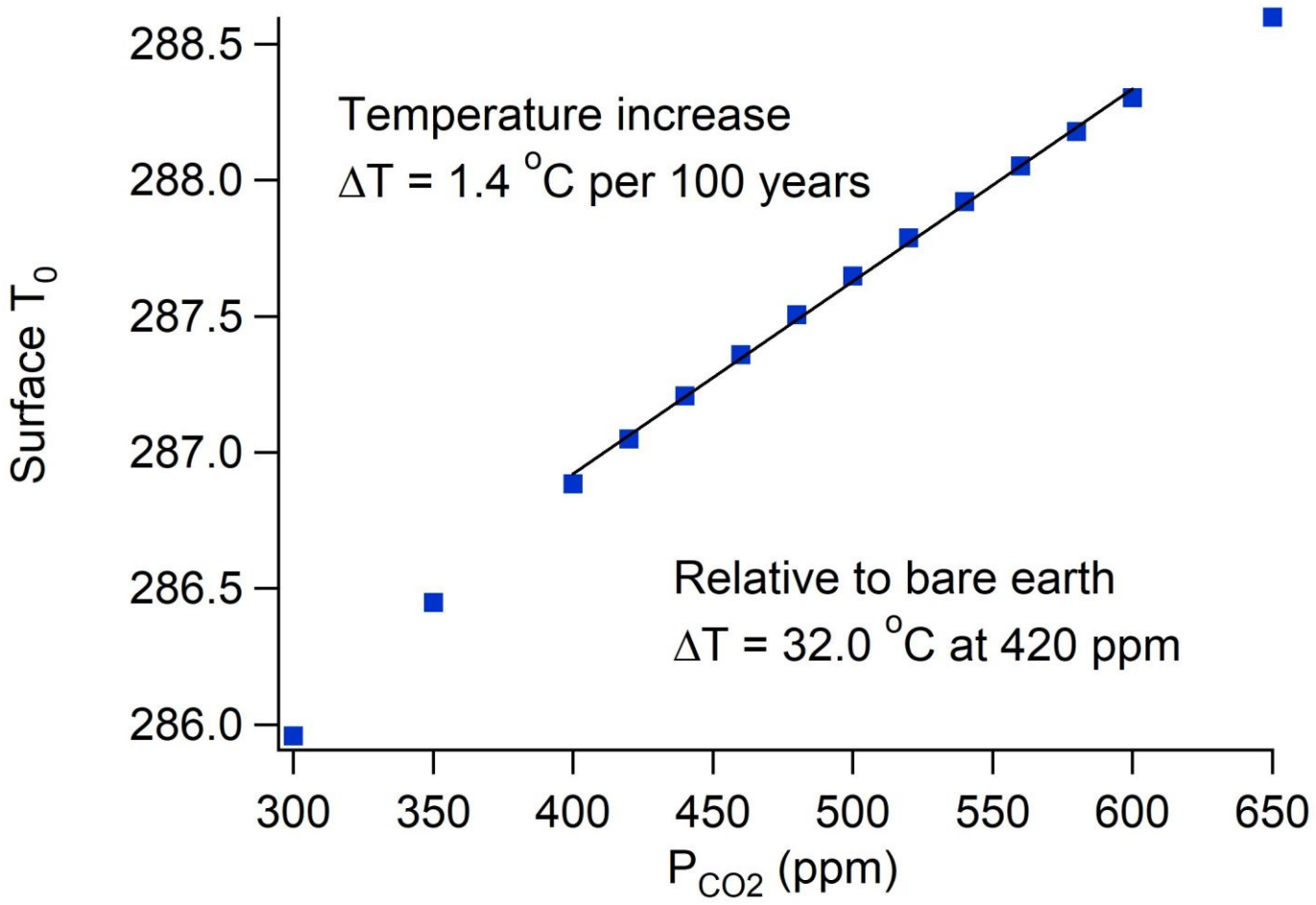
$$\tau_{atm} = \tau_{CO_2}\,\tau_{H_2O}$$

We use the approximate value of 0.80 for the transmittance of water.

Change in earth's surface temperature due to the CO_2 absorption

Greenhouse gas temperature rise at 410 ppm CO_2

$\Delta T = 32.0\ ^oC$



$\Delta T = 1.4\ ^oC$

Over 100 years for an annual Increase of 2.1 ppm