Temperature of planets at equilibrium

A. Calculate a realistic estimate for the temperature at the surface of Neptune using the black body radiation formula. Use the following facts.

Radius of Neptune: 24,746 km.
Distance of Neptune from the sun: 4.46 billion km.
The remaining needed data are available in Lecture 1.

Solution: Given data are:
$R_{Neptune} = 2.47 \times 10^7$ m
$R_{to\_sun} = 4.46 \times 10^{12}$ m

Given the radiant power of the sun at its surface is 3.2 x 1026 W as we calculated in class, the insolation on Neptune is given by this power divided by the area of a sphere at the radius equal to the distance from Neptune to the sun.

The area is $A = 4\pi R_{to\_sun}^2 = 4(3.14159)(4.46 \times 10^{12}\text{ m})^2 = 2.50 \times 10^{26}\text{ m}^2$.
The insolation is $I = 3.9 \times 10^{26}\text{ W}/2.58 \times 10^{26}\text{ m}^2 = 1.56\text{ W/m}^2$.
It is about 1/1000 as much flux as we receive here on earth.
Now we calculate the total power that Neptune absorbs. For this we need the cross sectional area of Neptune.
$A_{neptune\_cs} = \pi R_{Neptune}^2$
$2 = 3.14159(2.47 \times 10^7\text{ m})^2 = 1.91 \times 10^{15}\text{ m}^2$.
Thus the total power is:
$P_{abs} = IA_{neptune\_cs} = (1.56\text{ W/m}^2)(1.91 \times 10^{15}\text{ m}^2) = 2.98 \times 10^{15}$ W.
Now, we must reason that if Neptune is in equilibrium the power emitted as blackbody radiation must equal the power absorbed.
$P_{emit} = P_{abs}$
and
$P_{emit} = \sigma T_{Neptune}$
Ignoring the issue of the rotation of Neptune, we assume that radiation from the surface of Neptune is radiation from a sphere
$A_{Neptune} = 4\pi R_{Neptune}^2$
$A_{Neptune} = 4(3.14159)(2.47 \times 10^7\text{ m})^2 = 7.69 \times 10^{15}\text{ m}^2$.
$T_{Neptune} = (P_{abs} / A_{Neptune}/\sigma)^{1/4}$
$= (2.98 \times 10^{15}\text{ W}/7.69 \times 10^{15}\text{ m}^2/5.67 \times 10^{-8}\text{ kg s}^{-3}\text{ K}^{-4})^{1/4}$
$= 51.1$ K
Temperature = ____51.1 K________.

B. What wavelength is the peak of the black body emission from Neptune?

Solution:
$\lambda_{max}T = 2.897 \times 10^6$ nm-K
$\lambda_{max} = 2.897 \times 10^6\text{ nm-K}/T = 2.897 \times 10^6\text{ nm-K}/51.1\text{ K} = 24000\text{ nm} = 56.6\ \mu$

Wavelength = __56.6 microns or 56,600 nm___.