

Singular Value Decomposition

- Algorithm for factoring a matrix of multivariate data
- Creates a 2 new eigenvector matrices
- Diagonal matrix of scalar values
- Separates column vectors and row vectors
- Scales them with singular values
- Ranks them in order of decreasing importance

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 5 \\ \hline 4 & 5 & 5 \\ \hline 3 & 3 & 2 \\ \hline 4 & 5 & 4 \\ \hline 4 & 4 & 4 \\ \hline 3 & 5 & 4 \\ \hline 4 & 4 & 3 \\ \hline 2 & 4 & 4 \\ \hline 5 & 5 & 5 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 0.35 & 0.09 & -0.64 \\ \hline 0.38 & 0.19 & -0.10 \\ \hline 0.22 & -0.40 & 0.28 \\ \hline 0.36 & -0.08 & 0.33 \\ \hline 0.33 & -0.18 & -0.20 \\ \hline 0.33 & 0.33 & 0.48 \\ \hline 0.30 & -0.44 & 0.23 \\ \hline 0.28 & 0.64 & 0.10 \\ \hline 0.41 & -0.22 & -0.25 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 21.07 & 0 & 0 \\ \hline 0 & 2.01 & 0 \\ \hline 0 & 0 & 1.42 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 0.53 & 0.62 & 0.58 \\ \hline -0.82 & 0.20 & 0.53 \\ \hline -0.21 & 0.76 & -0.62 \\ \hline \end{array}$$

M matrix

U matrix

Σ matrix

VT matrix

How does SVD work?

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 5 \\ \hline 4 & 5 & 5 \\ \hline 3 & 3 & 2 \\ \hline 4 & 5 & 4 \\ \hline 4 & 4 & 4 \\ \hline 3 & 5 & 4 \\ \hline 4 & 4 & 3 \\ \hline 2 & 4 & 4 \\ \hline 5 & 5 & 5 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 0.35 & & \\ \hline 0.38 & & \\ \hline 0.22 & & \\ \hline 0.36 & & \\ \hline 0.33 & & \\ \hline 0.33 & & \\ \hline 0.30 & & \\ \hline 0.28 & & \\ \hline 0.41 & & \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 21.07 & 0 & 0 \\ \hline 0 & & 0 \\ \hline 0 & 0 & \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 0.53 & 0.62 & 0.58 \\ \hline & & \\ \hline & & \\ \hline \end{array}$$

M
U
Σ
VT

Factor Matrix, M, into Column Vectors, U, Singular Values, Σ, and Row Vectors, VT

$$\begin{array}{|c|c|c|} \hline 0.35 & & \\ \hline 0.38 & & \\ \hline 0.22 & & \\ \hline 0.36 & & \\ \hline 0.33 & & \\ \hline 0.33 & & \\ \hline 0.30 & & \\ \hline 0.28 & & \\ \hline 0.41 & & \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 21.07 & 0 & 0 \\ \hline 0 & & 0 \\ \hline 0 & 0 & \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 0.53 & 0.62 & 0.58 \\ \hline & & \\ \hline & & \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 3.95 & 4.64 & 4.34 \\ \hline 4.27 & 5.02 & 4.69 \\ \hline 2.42 & 2.85 & 2.66 \\ \hline 3.97 & 4.67 & 4.36 \\ \hline 3.64 & 4.28 & 4.00 \\ \hline 3.69 & 4.33 & 4.05 \\ \hline 3.33 & 3.92 & 3.66 \\ \hline 3.08 & 3.63 & 3.39 \\ \hline 4.55 & 5.35 & 5.00 \\ \hline \end{array}$$

\neq

$$\begin{array}{ccc|ccc}
 4 & 4 & 5 & 3.95 & 4.64 & 4.34 \\
 4 & 5 & 5 & 4.27 & 5.02 & 4.69 \\
 3 & 3 & 2 & 2.42 & 2.85 & 2.66 \\
 4 & 5 & 4 & 3.97 & 4.67 & 4.36 \\
 4 & 4 & 4 & 3.64 & 4.28 & 4.00 \\
 3 & 5 & 4 & 3.69 & 4.33 & 4.05 \\
 4 & 4 & 3 & 3.33 & 3.92 & 3.66 \\
 2 & 4 & 4 & 3.08 & 3.63 & 3.39 \\
 5 & 5 & 5 & 4.55 & 5.35 & 5.00
 \end{array}
 -
 \begin{array}{ccc|ccc}
 3.95 & 4.64 & 4.34 & 0.05 & -0.64 & 0.66 \\
 4.27 & 5.02 & 4.69 & -0.28 & -0.02 & 0.31 \\
 2.42 & 2.85 & 2.66 & 0.58 & 0.15 & -0.66 \\
 3.97 & 4.67 & 4.36 & 0.03 & 0.33 & -0.36 \\
 3.64 & 4.28 & 4.00 & 0.36 & -0.28 & 0.00 \\
 3.69 & 4.33 & 4.05 & -0.69 & 0.67 & -0.05 \\
 3.33 & 3.92 & 3.66 & 0.67 & 0.08 & -0.66 \\
 3.08 & 3.63 & 3.39 & -1.08 & 0.37 & 0.61 \\
 4.55 & 5.35 & 5.00 & 0.45 & -0.35 & 0.00
 \end{array}
 =
 \begin{array}{ccc|ccc}
 0.05 & -0.64 & 0.66 & & & \\
 -0.28 & -0.02 & 0.31 & & & \\
 0.58 & 0.15 & -0.66 & & & \\
 0.03 & 0.33 & -0.36 & & & \\
 0.36 & -0.28 & 0.00 & & & \\
 -0.69 & 0.67 & -0.05 & & & \\
 0.67 & 0.08 & -0.66 & & & \\
 -1.08 & 0.37 & 0.61 & & & \\
 0.45 & -0.35 & 0.00 & & &
 \end{array}$$

← Difference Matrix

- Subtracts the factored matrix from the original matrix
- Creates a difference matrix
- Factors the difference matrix
- Obtain second column vector, singular value, and row vector

$$\begin{array}{ccc|ccc}
 0.05 & -0.64 & 0.66 & & & \\
 -0.28 & -0.02 & 0.31 & & & \\
 0.58 & 0.15 & -0.66 & & & \\
 0.03 & 0.33 & -0.36 & & & \\
 0.36 & -0.28 & 0.00 & \neq & & \\
 -0.69 & 0.67 & -0.05 & & & \\
 0.67 & 0.08 & -0.66 & & & \\
 -1.08 & 0.37 & 0.61 & & & \\
 0.45 & -0.35 & 0.00 & & &
 \end{array}
 *
 \begin{array}{ccc|ccc}
 & & & 0.09 & & \\
 & & & 0.19 & & \\
 & & & -0.40 & & \\
 & & & -0.08 & & \\
 & & & -0.18 & & \\
 & & & 0.33 & & \\
 & & & -0.44 & & \\
 & & & 0.64 & & \\
 & & & -0.22 & &
 \end{array}
 *
 \begin{array}{ccc|ccc}
 & & & 0 & 0 & \\
 0 & 2.01 & 0 & & & \\
 0 & 0 & & & &
 \end{array}
 *
 \begin{array}{ccc|ccc}
 & & & -0.82 & 0.20 & 0.53 \\
 & & & & & \\
 & & & & &
 \end{array}$$

$$\begin{array}{|c|c|c|} \hline 4 & 4 & 5 \\ \hline 4 & 5 & 5 \\ \hline 3 & 3 & 2 \\ \hline 4 & 5 & 4 \\ \hline 4 & 4 & 4 \\ \hline 3 & 5 & 4 \\ \hline 4 & 4 & 3 \\ \hline 2 & 4 & 4 \\ \hline 5 & 5 & 5 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 0.35 & 0.09 & -0.64 \\ \hline 0.38 & 0.19 & -0.10 \\ \hline 0.22 & -0.40 & 0.28 \\ \hline 0.36 & -0.08 & 0.33 \\ \hline 0.33 & -0.18 & -0.20 \\ \hline 0.33 & 0.33 & 0.48 \\ \hline 0.30 & -0.44 & 0.23 \\ \hline 0.28 & 0.64 & 0.10 \\ \hline 0.41 & -0.22 & -0.25 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 21.07 & 0 & 0 \\ \hline 0 & 2.01 & 0 \\ \hline 0 & 0 & 1.42 \\ \hline \end{array} * \begin{array}{|c|c|c|} \hline 0.53 & 0.62 & 0.58 \\ \hline -0.82 & 0.20 & 0.53 \\ \hline -0.21 & 0.76 & -0.62 \\ \hline \end{array}$$

M matrix

U matrix

Σ matrix

VT matrix

Now this overall statement is true.