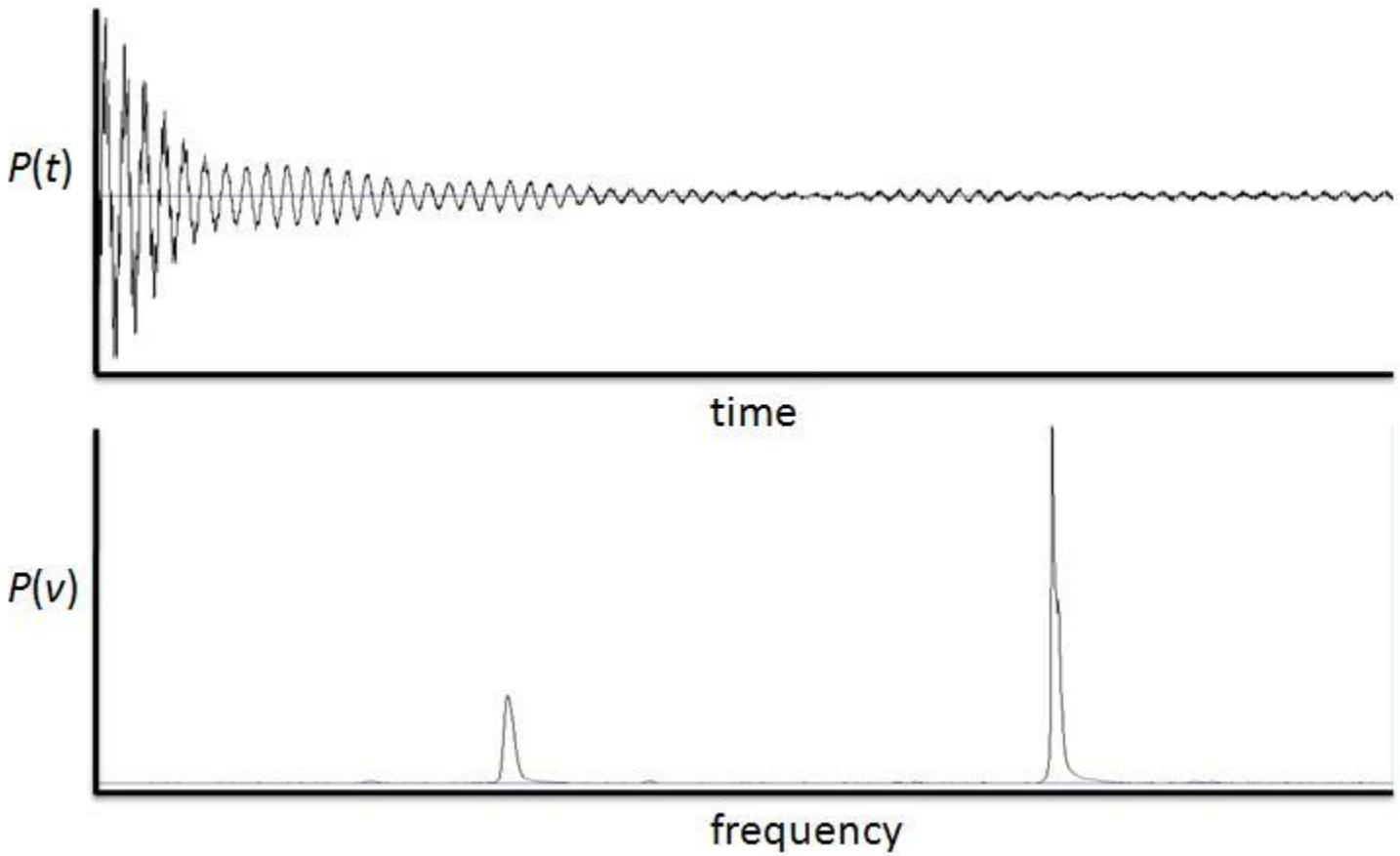
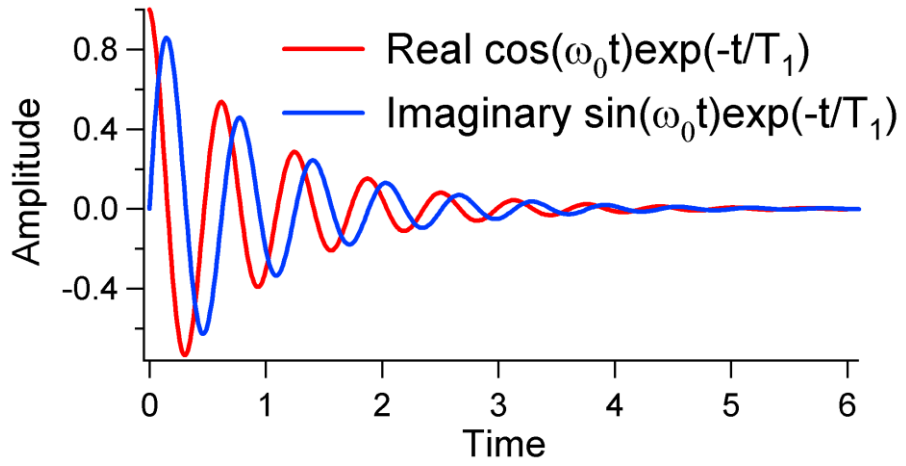
Classic example: Free induction decay in NMR



The Free Induction Decay

One example of a common Fourier transform is the free induction decay in NMR. In this instance there are both real and imaginary parts. This is because when the magnetization is rotated into the x-y plane it is detected by Coils along both x and y (out-of-phase and in-phase).

$$L(\omega) = \int_{-\infty}^{\infty} exp\{-i(\omega - \omega_0)t - \Gamma t\}$$



Lorentzian broadening

A Lorentzian is the Fourier-transform of an exponential function. To see this we begin with the definition of a Fourier transform.

$$L(\omega) = \frac{1}{\pi}\int_0^\infty f(t)e^{i\omega t - \Gamma t}dt$$

We then substitute in $e^{-\Gamma t}$ where $\Gamma = 1/T_2$ is the relaxation time of the excited state. The precise model for the excited state dynamics depends on the system. If we consider magnetic resonance, we will include magnetic inhomogeneity in the linewidth and so T_2 is used. However, for optical transitions the homogeneous line width may include only the excited state life time, or natural life time, T_1.

The Fourier transform

This expression includes the appropriate normalization constant for a Lorentzian. In the general case of a Lorentzian centered about ω_o, we have

$$\frac{1}{\pi}\int_0^\infty e^{i(\omega-\omega_o)t-\Gamma t}dt = \frac{1}{\pi}\frac{1}{i(\omega-\omega_o)-\Gamma}$$

$$= \frac{1}{\pi}\frac{1}{i(\omega-\omega_o)-\Gamma}\left(\frac{-i(\omega-\omega_o)-\Gamma}{-i(\omega-\omega_o)-\Gamma}\right)$$

Absorption and dispersion

Out-of-phase
Absorption

In-phase
Dispersion

$$= \frac{1}{\pi}\frac{\Gamma}{(\omega-\omega_o)^2+\Gamma^2} - \frac{i}{\pi}\frac{(\omega-\omega_o)}{(\omega-\omega_o)^2+\Gamma^2}$$

The resulting function is complex. One can think of a complex function as resulting from in-phase and out-of-phase terms. An in-phase term leads to dispersion of the light and and out-of-phase term leads to absorption.

Absorption and dispersion line shapes