## Linear Regression Standard error of a slope

From RLS to Trumpets

RLS = Robust Least Squares

Trumpets = Calculate Upper and Lower 95% Confidence Limits

There are two regression worksheets: RLS and Trumpets

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Week 6 - Lab	Introduction to Statistics - (Intro, Error types, Physical measurements and	V Quizzes								



# Let's follow the download of the trumpets



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### Trumpets: the confidence limits of a line

The calibration error can statistically be represented by drawing the 95% **confidence** limits around the calibration line. These limits form the two branches of a hyperbolic function. The two sets of hyperbolic branches are given by:

$$Y_{confidence} = sX + b \pm t RMSE \sqrt{\frac{1}{N} + \frac{N(X - \bar{X})^2}{N\sum X^2 - (\sum X)^2}}$$

These are the trumpets. The t is determined using the t-test.

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The mean is the value of interest and upper – lower = 95% Confidence Limit

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It is normally the case that the limit of detection (LOD) is larger than the error in the mid-regression. The errors in the ends are not as well determined since there are fewer data points. Read off the upper and lower 95% to 4 sig figs.

Calculate the difference and this is the 95% Confidence interval

#### for the calibration at this value.

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