Continuous distribution

Normal distribution

Central limit theorem

The Gaussian function

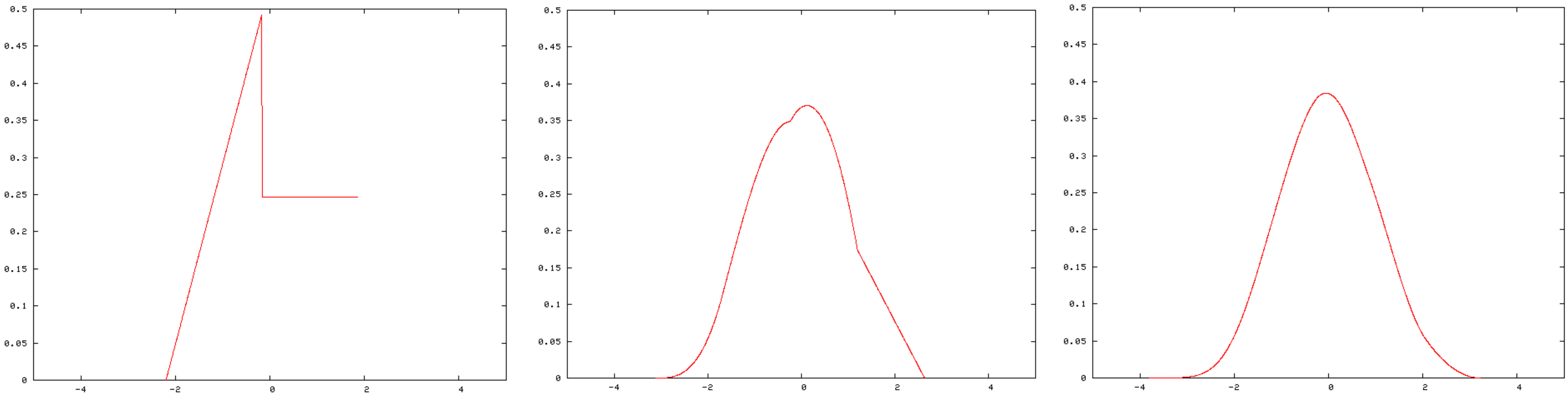
What is the distribution of continuous probabilities?

The distribution of continuous probabilities will also be approximately Gaussian. Random fluctuations will tend to cluster near the average (mean).

We call the distribution of the random errors a normal distribution. It is given by a Gaussian function according to the central limit theorem.

Just as for the discrete variable, the approach to a Gaussian is clearer to more data we obtain.

The normal distribution is approached in the limit

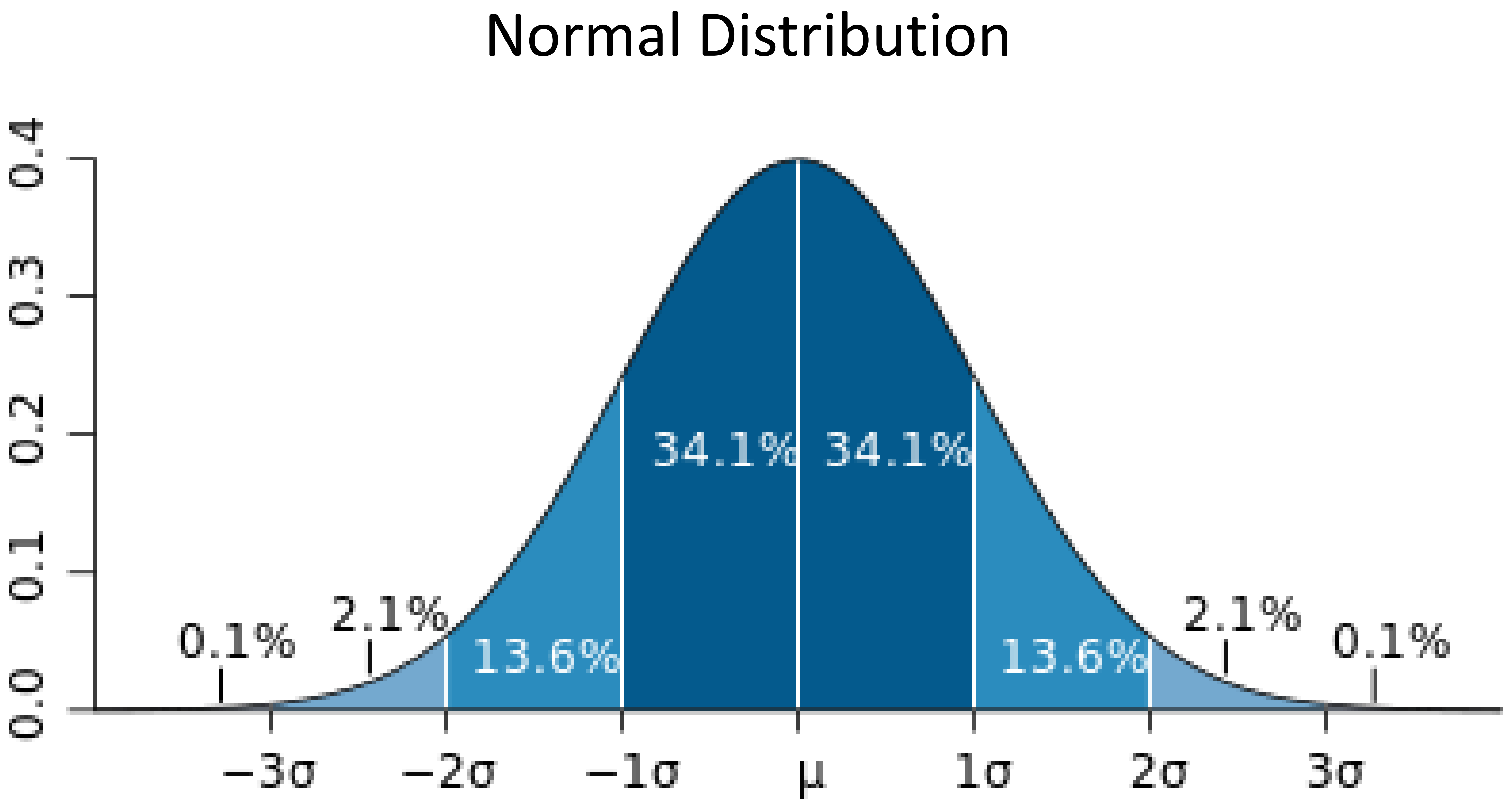


Increasing number of measurements

This is known as the central limit theorem

For data or observations that contain random noise the distribution will approach a normal (Gaussian) distribution as the number of observations approaches infinity.

A Gaussian function describes a normal distribution



Properties of a Gaussian function

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} exp\left\{-\frac{(x-\langle x\rangle)^2}{2\sigma^2}\right\}$$

Case of a discrete random variable distribution:

Least squares definitions

The mean is <x>

$$\mu_{LS} = \langle x \rangle = \frac{\sum_i x_i}{N}$$

The variance is σ^2/N

The standard deviation is $\sigma_{LS} = \sqrt{\frac{\sum_i (x_i - \langle x\rangle)^2}{N}}$

Standard deviation and standard error

We can call the variance the standard error.

The standard deviation is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \langle x \rangle)^2}{N}}$$

And the standard error is:

$$\sigma_{se} = \frac{\sqrt{\sum_{i=1}^{n}(x_i - \langle x \rangle)^2}}{N} = \frac{\sigma}{\sqrt{N}}$$