Properties of a group

1. There must exist an identity operator which commutes with all other operators.
2. The product of any two operators must also be a member of the group.
3. Multiplication is associative, but not necessarily commutative.
4. There must exist an inverse (or reciprocal) for each element in the group.

Corollaries:

1. The identity operator is its own inverse.
2. A similarity transform is an operation:

$$Z^{-1}XZ = Y$$

Point groups

We can assemble the operations of the group into a multiplication table. This group of operations satisifes all of the requirements of a mathematical group and is called a point group. Point groups get their name from the fact that at least one point in space remains unchanged for all operations in the group.

C_1 is a point group whose only symmetry operation is E, the identity. In other words there is no symmetry.

C_s is a point group whose symmetry operations are E and σ. The symmetry is restricted to a mirror plane.

Point group examples C_1 and Cs

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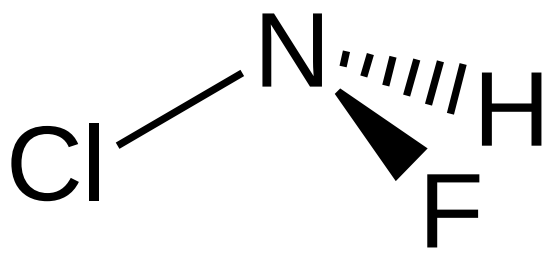
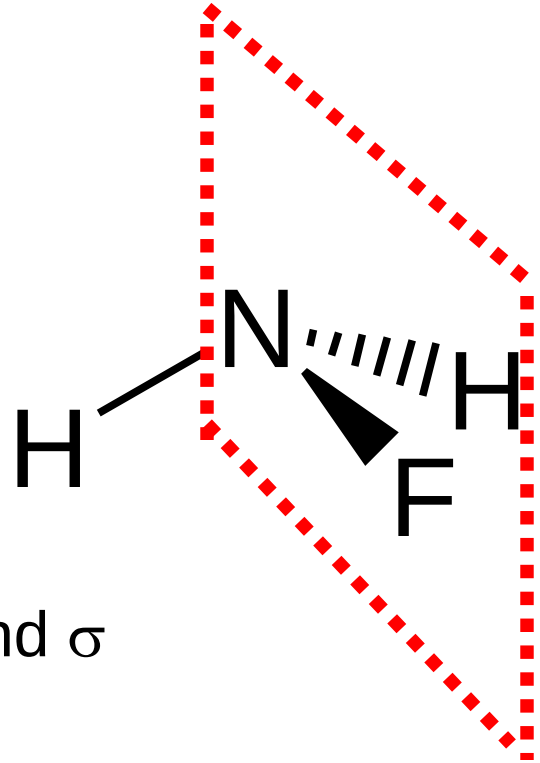
Point group examples C_1 and C_s

C_1



E only

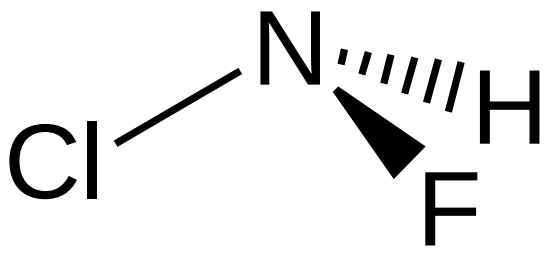
C_s



E and σ

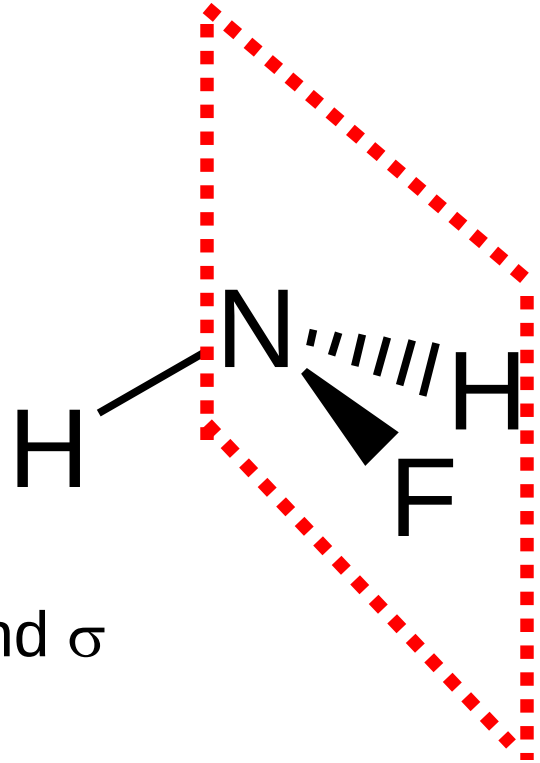
Point group examples C_1 and C_s

C_1



E only

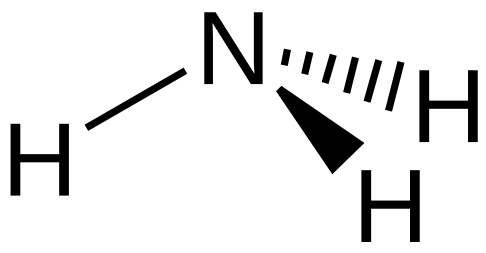
C_s



E and σ

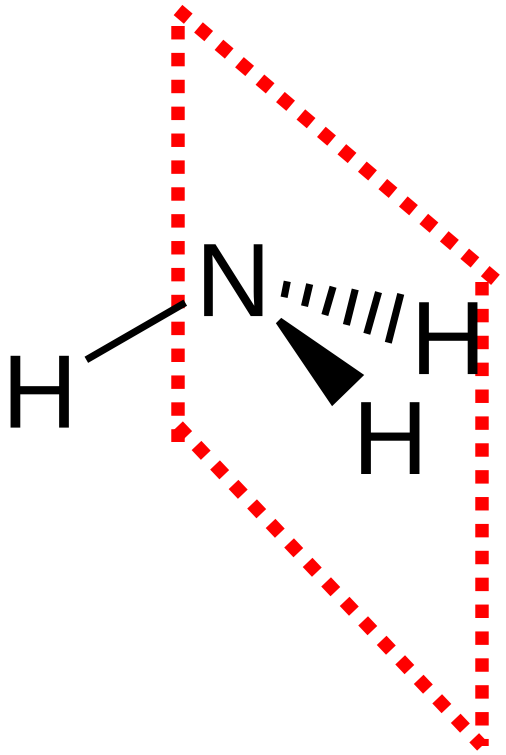
C_1	Symmetry elements for the group	Spectroscopy active component		
	E	Microwave	IR	Raman
A	1	(none)		

Point group example: Ammonia C_{3v}



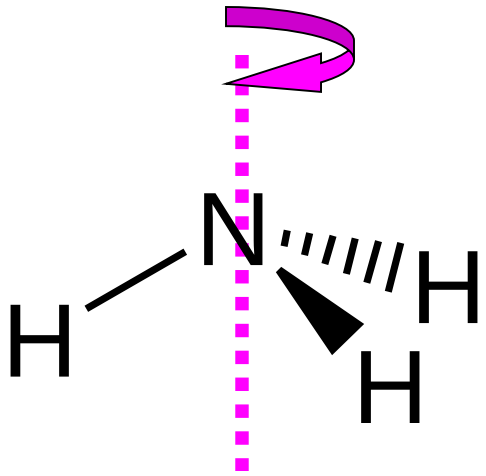
The symmetry operation E exists for all groups.

Point group example: Ammonia C_{3v}



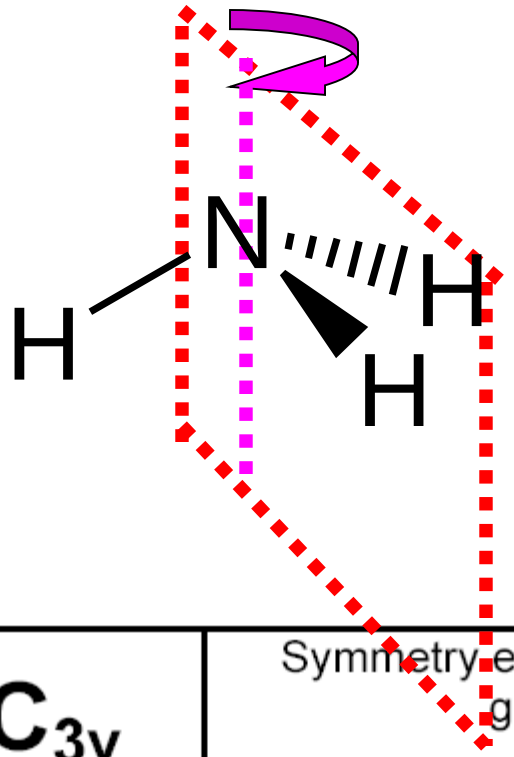
A vertical reflection plane σ_v is shown. There are three such planes in molecules in the C_{3v} point group.

Point group example: Ammonia C_{3v}



There are two possible Rotations about a 3-fold axis. The first is a 120^{o} rotation and the second is a 240^{o} rotation.

Point group example: Ammonia C_{3v}



The group consists of these three symmetry operations. The order of the group is h=6. There are three irreducible representations in the point group C_{3v}, which are given in the character table below.

C_{3v}		Symmetry elements for the group			Spectroscopy active component		
		E	$2C_3$	$3s_v$	Microwave	IR	Raman
Symmetry label	A_1	1	1	1		z	$x^2 + y^2, z^2$
	A_2	1	1	-1	R_z		
	E	2	-1	0	R_y, R_x	x, y	$(x^2 + y^2, xy)$ (xz, yz)