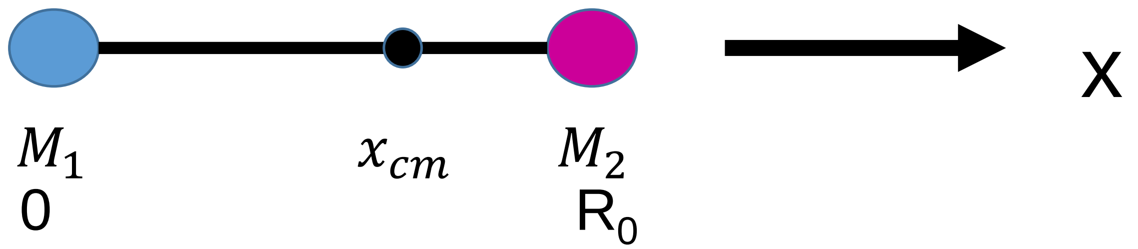
Center of mass and reduced mass problem

Given the drawing below determine the quantities in terms of M_1 , M_2 and R_0. Please assume that the origin is at mass 1 and the distance R_0 is at mass 2.

(a) the center of mass along the x axis as a fraction of R_0.



(b) the definition of the reduced mass μ based on the moment of inertia

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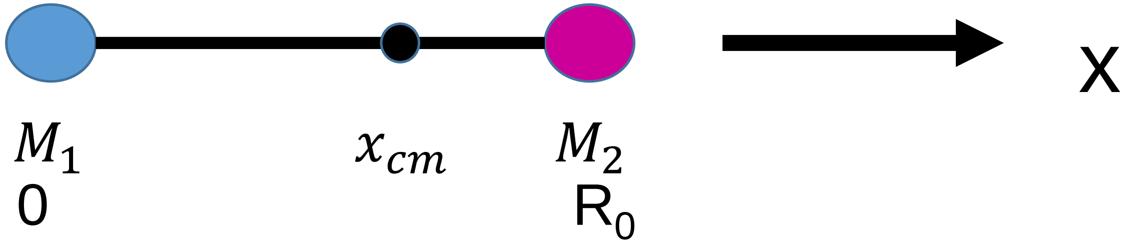

The center of mass is given by:

$$\sum_{i=1}^{N} m_i r_i = 0$$

$$-M_1 x_{cm} + M_2(R_0 - x_{cm}) = 0$$

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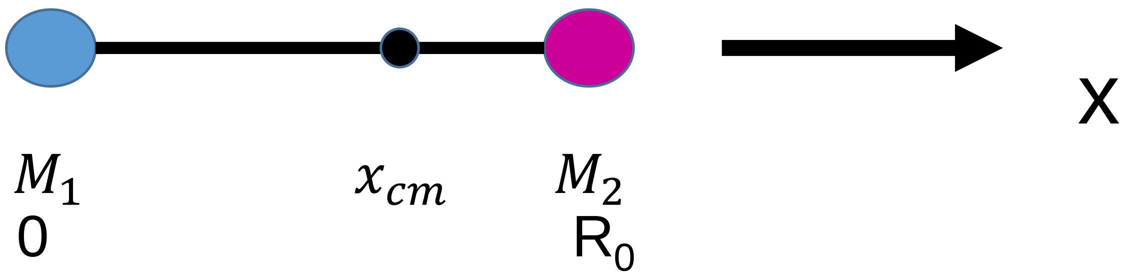

Solving for x_{cm}

$$-M_1 x_{cm} + M_2(R_0 - x_{cm}) = 0$$

The center of mass is given by:

$$x_{cm} = \frac{M_2}{M_1 + M_2} R_0$$

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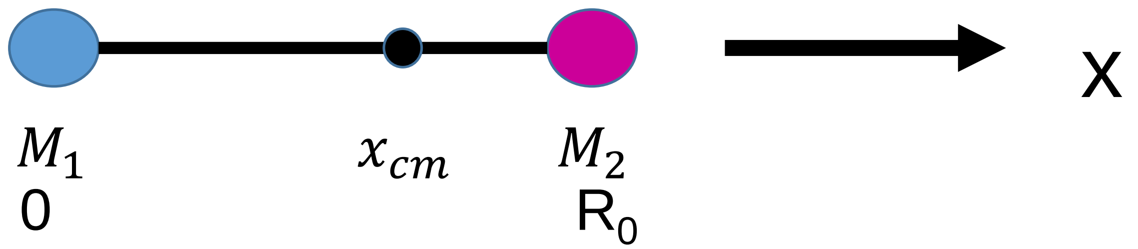

(b) the definition of the reduced mass μ based on the moment of inertia

The moment of inertia for both masses is

$$I = \sum_{i=1}^{N} m_i r_i^2$$

$$I = M_1 x_{cm}^2 + M_2 (R_0 - x_{cm})^2$$

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Substituting in for the center of mass

$$I = M_1 \left(\frac{M_2}{M_1 + M_2}\right)^2 R_0^2 + M_2 \left(R_0 - \frac{M_2}{M_1 + M_2} R_0\right)^2$$

Center of mass and reduced mass problem

And carrying out the algebra on the mass terms

$$I = M_1 \left(\frac{M_2}{M_1 + M_2} \right)^2 R_0^2 + M_2 \left(1 - \frac{M_2}{M_1 + M_2} \right)^2 R_0^2$$

$$I = M_1 \left(\frac{M_2}{M_1 + M_2} \right)^2 R_0^2 + M_2 \left(\frac{M_1}{M_1 + M_2} \right)^2 R_0^2$$

$$I = \left(\frac{M_1 M_2^2}{(M_1 + M_2)^2} + \frac{M_2 M_1^2}{(M_1 + M_2)^2} \right) R_0^2$$

Center of mass and reduced mass problem

Collecting terms we have

$$I = \left(\frac{M_1 M_2 (M_1 + M_2)}{(M_1 + M_2)^2}\right) R_0^2$$

And dividing we obtain

$$I = \left(\frac{M_1 M_2}{M_1 + M_2}\right) R_0^2$$

which can be compared to the required definition

$$I = \mu R_0^2$$

Center of mass and reduced mass problem

Comparing

$$I = \left(\frac{M_1 M_2}{M_1 + M_2}\right) R_0^2$$

to

$$I = \mu R_0^2$$

We see that

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$