Normalization of a harmonic oscillator wave function

We have discussed the normalization of the v = 0 harmonic oscillator wave function in class. In this problem you are asked to determine the normalization constant for the v = 1 harmonic oscillator wave function. Given that the wave function is:

$$\chi_1 = N_1 2\sqrt{\alpha} Q e^{-\frac{\alpha Q^2}{2}}$$

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Given that the wave function is:

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Set up the normalization problem by squaring the wave function

$$1 = \int_{-\infty}^{\infty} \chi_1 \, \chi_1 dQ = N_1^2 \int_{-\infty}^{\infty} 2\sqrt{\alpha} Q e^{-\alpha Q^2/2} \, 2\sqrt{\alpha} Q e^{-\frac{\alpha Q^2}{2}} dQ$$

Note that the Hermite functions are real so we do not need to worry about the complex conjugate of the wave function.

$$1 = N_1^2 4\alpha \int_{-\infty}^{\infty} 4\alpha Q^2 e^{-\alpha Q^2} \, dQ$$

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Given that the wave function is:

$$\chi_1 = N_1 2\sqrt{\alpha} Q e^{-\frac{\alpha Q^2}{2}}$$

We use the Gaussian polynomial formula:

$$1 = N_1^2 4\alpha(-1)^1 \frac{\partial}{\partial \alpha}\left(\frac{\pi}{\alpha}\right)^{1/2} = N_1^2 4\alpha \frac{1}{2\alpha}\left(\frac{\pi}{\alpha}\right)^{1/2}$$

To obtain the square of the normalization constant.

$$N_1^2 = \frac{1}{2}\left(\frac{\alpha}{\pi}\right)^{1/2}$$

Finally, we take the square root to find the normalization constant:

$$N_1 = \frac{1}{\sqrt{2}}\left(\frac{\alpha}{\pi}\right)^{1/4}$$