Rotational transition moment

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Solution: the general solution for rotation states is

$$Y_{JM}(\theta,\phi) = \sqrt{\frac{(2J+1)(J-M)!}{4\pi(J+M)!}} P_{JM}(cos\theta)e^{iM\phi}$$

For J = 0, M = 0

$$Y_{00}(\theta,\phi) = \sqrt{\frac{1}{4\pi}}$$

For J = 1, M = 0

$$Y_{10}(\theta,\phi) = \sqrt{\frac{(2+1)(1)!}{4\pi(1)!}} P_{10}(cos\theta) = \sqrt{\frac{3}{4\pi}} cos\theta$$

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The transition moment is:

$$M_{01} = \int_0^{2\pi} d\phi \int_{-1}^{1} Y_{10}(\theta,\phi)\mu_z x Y_{00}(\theta,\phi)dx$$

where we make the substitution x = cosθ.

$$M_{01} = 2\pi \int_{-1}^{1} \sqrt{\frac{3}{4\pi}}\, x\mu_z x \sqrt{\frac{1}{4\pi}}\, dx$$

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Consolidating terms we find that the transition dipole moment is

$$M_{01} = \sqrt{\frac{3}{4}}\mu_z \int_{-1}^{1} x^2 dx = \sqrt{\frac{3}{4}}\mu_z \left(\frac{1}{3} - \frac{-1}{3}\right) = \frac{\mu_z}{\sqrt{3}}$$