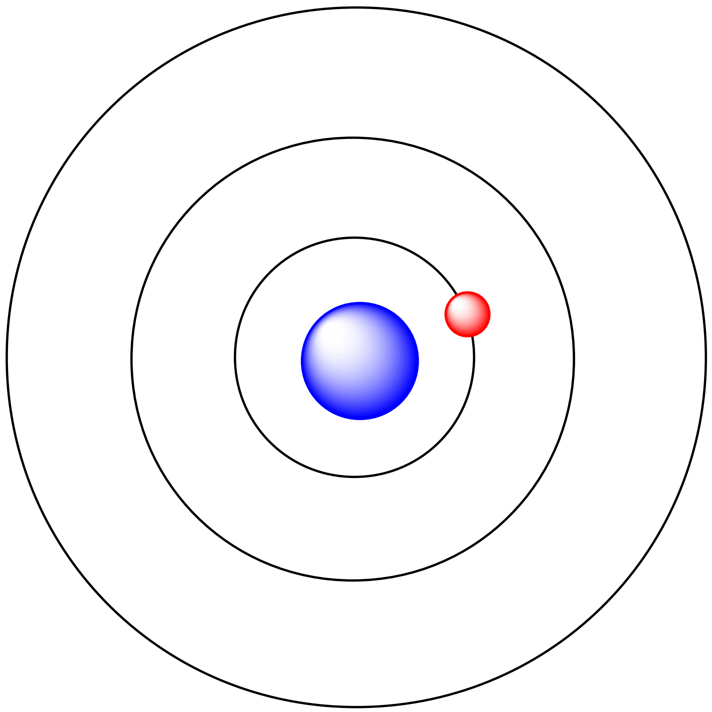
Bohr model for hydrogen

Planetary model for hydrogen

Niels Bohr observed that the Rydberg formula could be explained if one assumed that there were electronic "orbits" around the hydrogen atom nucleus. Bohr came to this conclusion by starting with classical physics using a "planetary model" for the hydrogen atom.



Balance of forces required to maintain orbit

Imagine that the electron orbits the nucleus in the same manner that a planet orbits the Sun. Instead of a gravitational attraction, the electron and the proton in the nucleus have a Coulombic attraction, which balances the centripetal force according to,

$$f = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

The term $4\pi\varepsilon_0$ is nothing more than a unit conversion factor that keeps the force in Newtons.

ε_0 is called the permittivity of vacuum and it has the value

$\varepsilon_0 = 8.854 \times 10^{-12}\ J^{-1}m^{-1}C^2$.

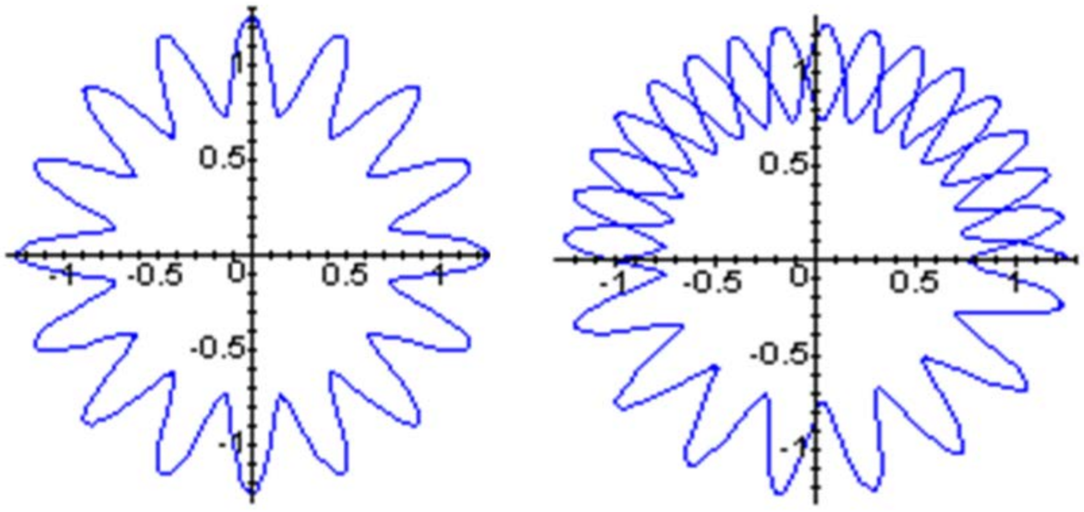
Wave picture for the circular orbit

In to use this classical expression for the circular orbit, Bohr followed the Planck and DeBroglie idea that the electron behaved like a wave. Therefore, the electron is required to have an integral number of wavelengths around its circular orbit.

If n is not an integer then the electron interferes with itself. The result is:

$$2\pi r = n\lambda$$

This is one of two simple ideas that led to the Bohr model for hydrogen.



n is an integer

n is not an integer

Use DeBroglie relation to quantize angular momentum

The second idea was to combine this result with the DeBroglie relation

$$2\pi r = \frac{nh}{p} = \frac{nh}{mv}$$

The angular momentum J = mvr is then quantized according to

$$mvr = \frac{nh}{2\pi} = n\hbar$$

Putting these results together we find

$$\frac{me^2}{4\pi\varepsilon_0} = \frac{m^2v^2r^2}{r} = \frac{n^2\hbar^2}{r}$$

The Bohr radius

These equations imply that the radius is quantized. The radius with n = 1 is known as the Bohr radius.

$$r = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{me^2}$$

This equation predicts that the orbits around the nucleus are quantized, by an integer, n. According to classical electrostatics the energy of the electron is composed of kinetic energy, T and potential energy V, given by,

$$E = T + V = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Bohr’s model obtained the Rydberg constant

Substituting in the value of r, we obtain.

$$E_n = -\frac{me^4}{8\varepsilon_0{}^2h^2}\frac{1}{n^2}$$

where the value of the constants gives the Rydberg constant to relatively high accuracy.

$$R = \frac{me^4}{8\varepsilon_0{}^2h^2}$$