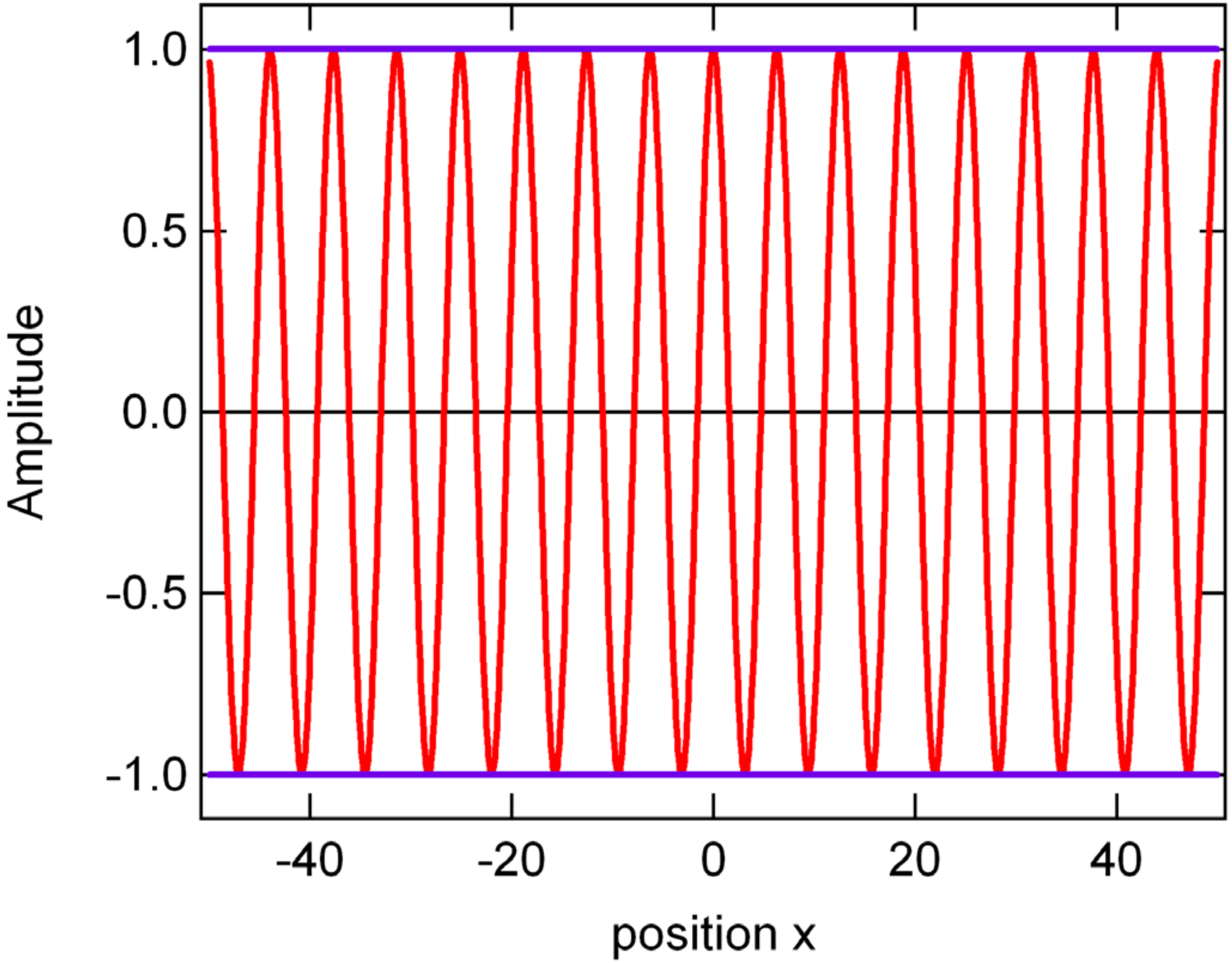
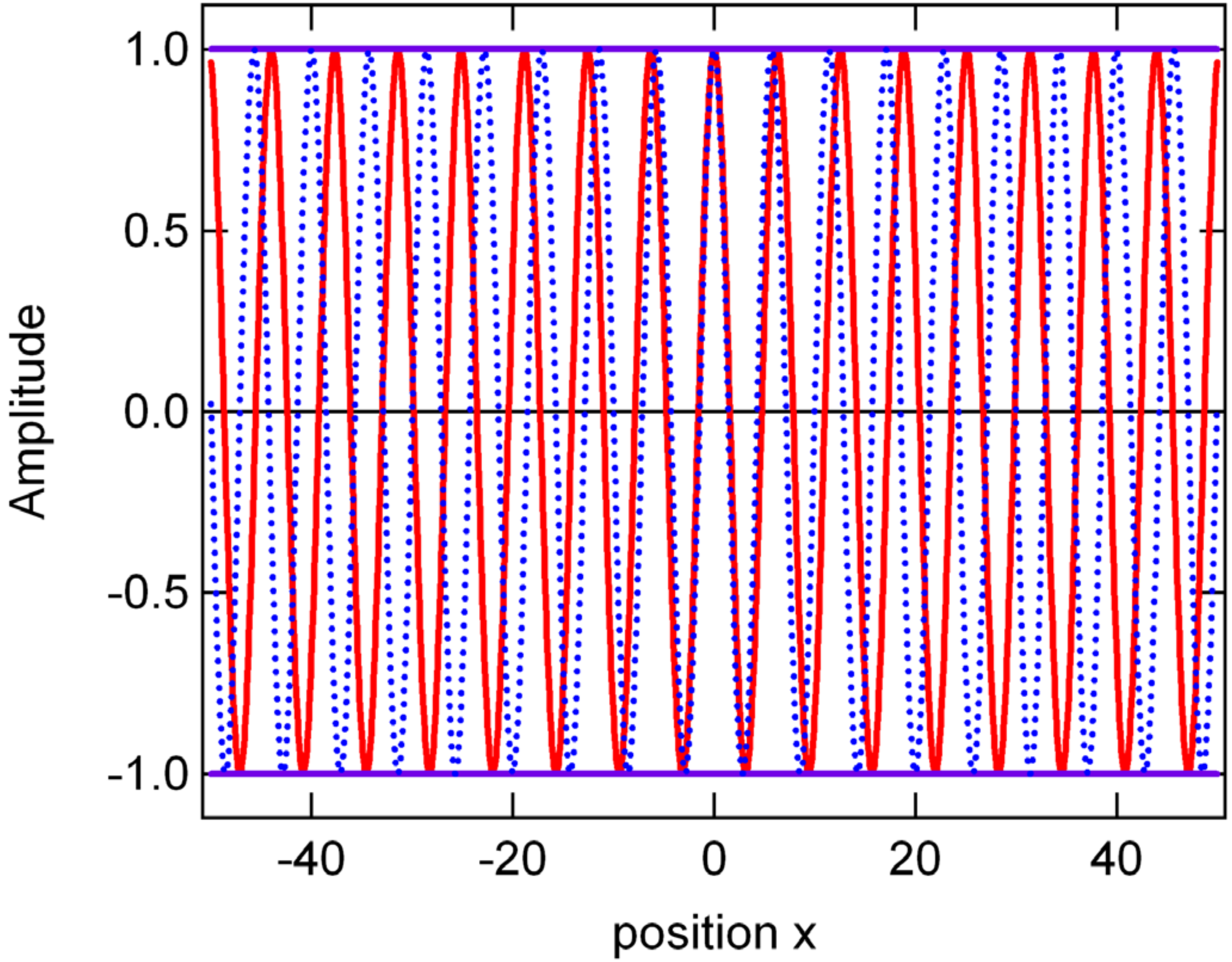
Particle location in free space is not defined

$$\Psi(x) = \sin(kx)$$



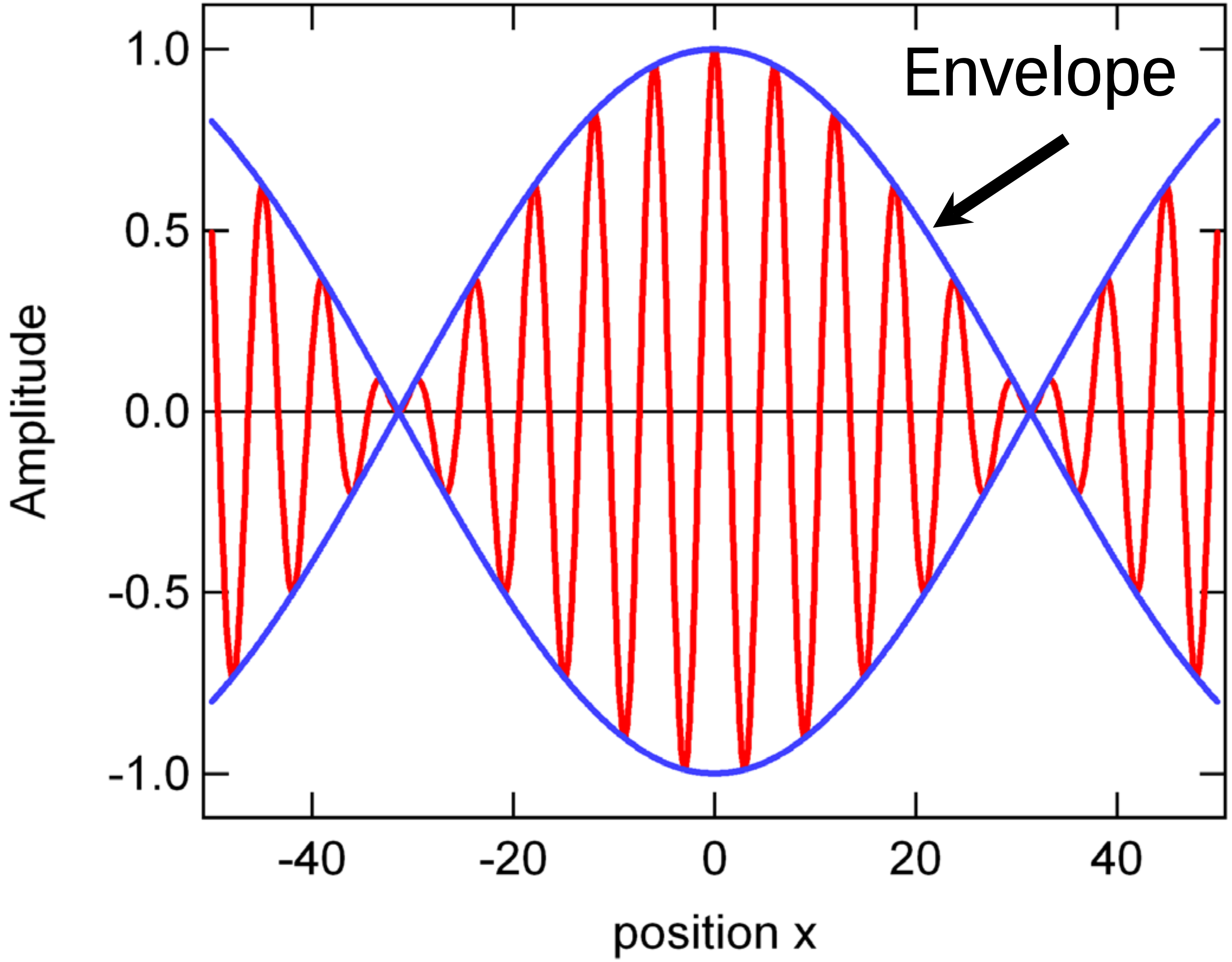
Superposition wave with momentum hk and h(1.1k)

$$\Psi(\mathrm{x}) = \sin(kx)\ and\ \Psi(\mathrm{x}) = \sin(1.1kx)$$



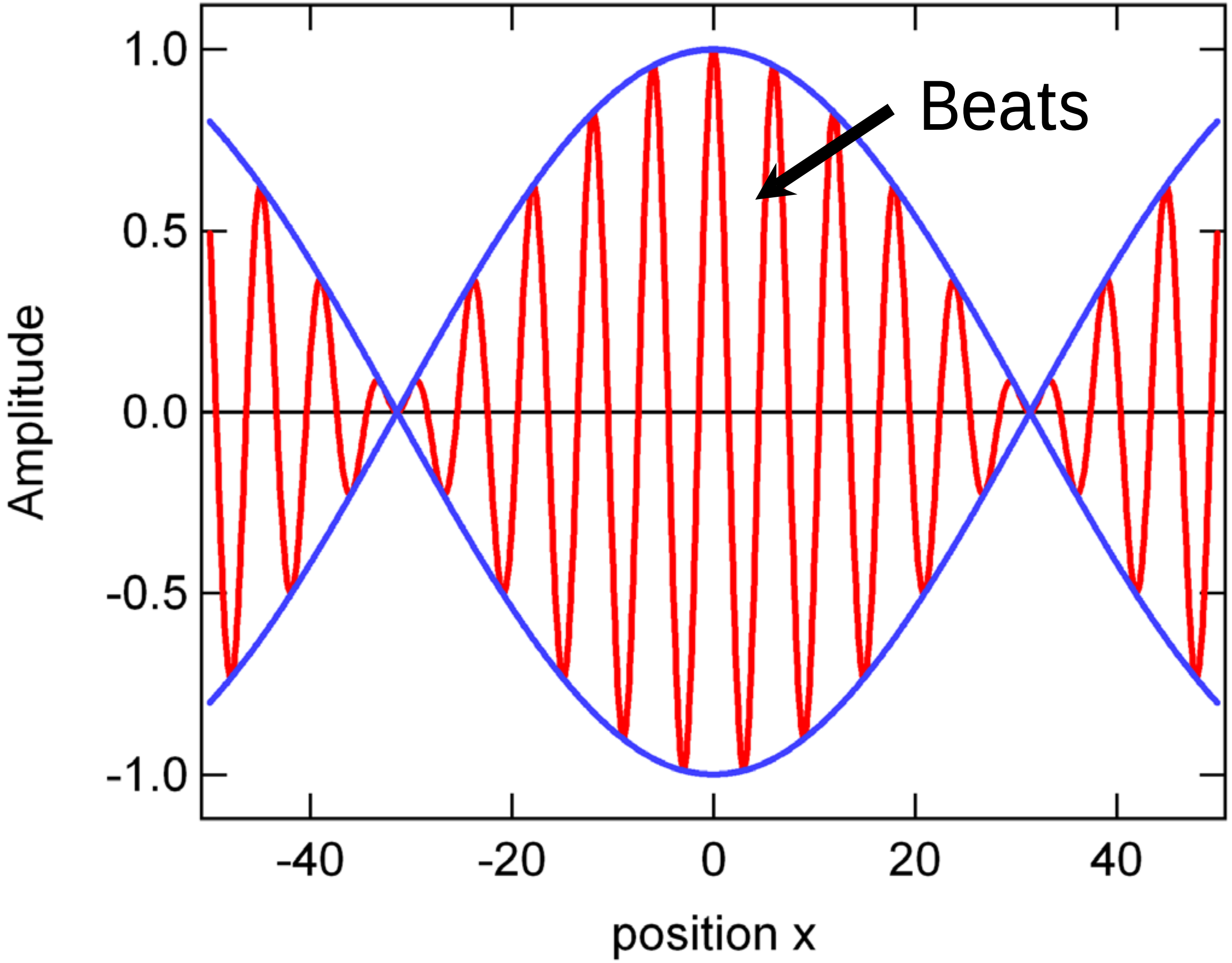
Envelope frequency at $(k_2 - k_1)/2$

$$\Psi(\mathrm{x}) = \sin(kx) + \sin(1.1kx)$$



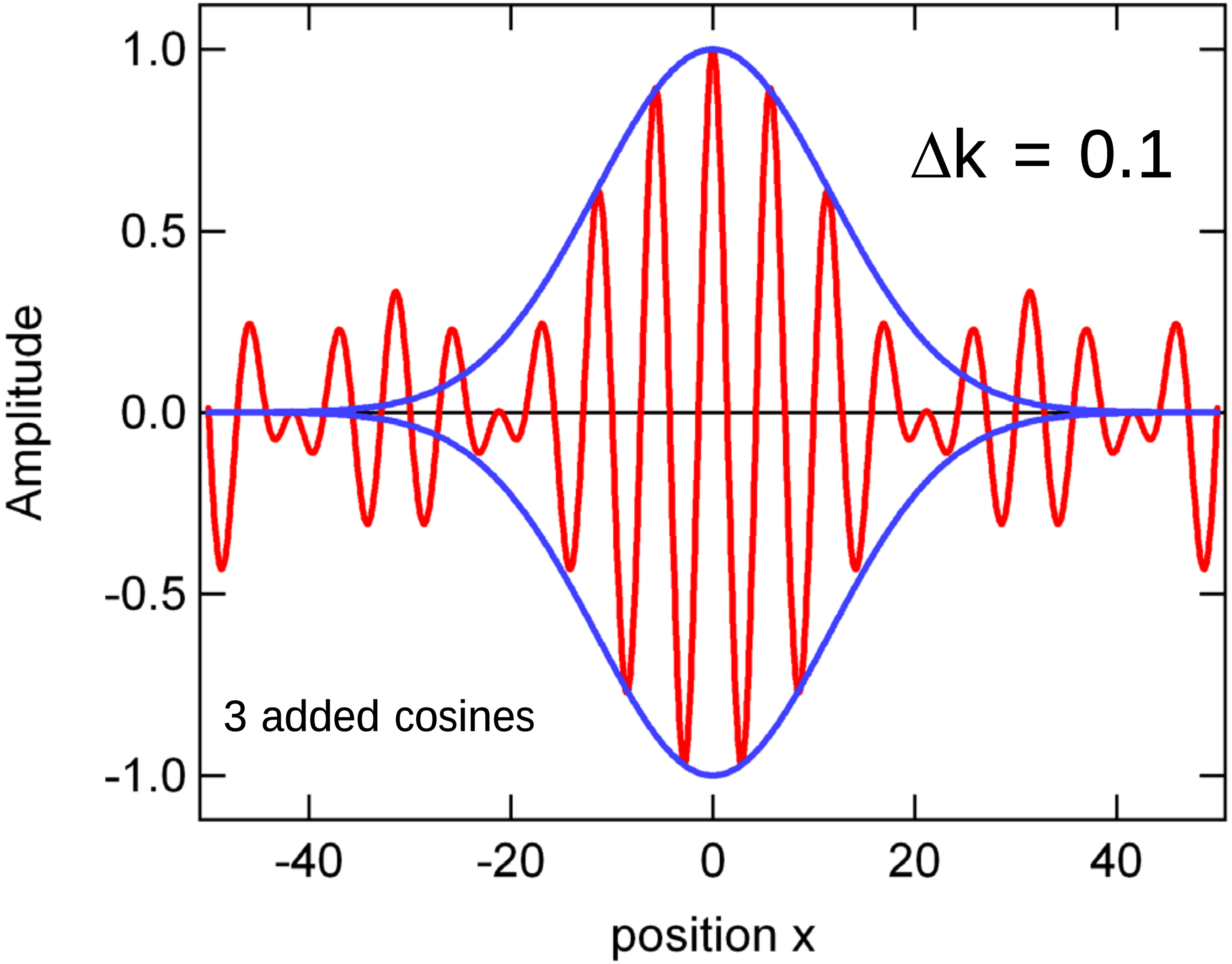
Beat frequency at $(k_2 + k_1)/2$

$$\Psi(\mathrm{x}) = \sin(kx) + \sin(1.1kx)$$



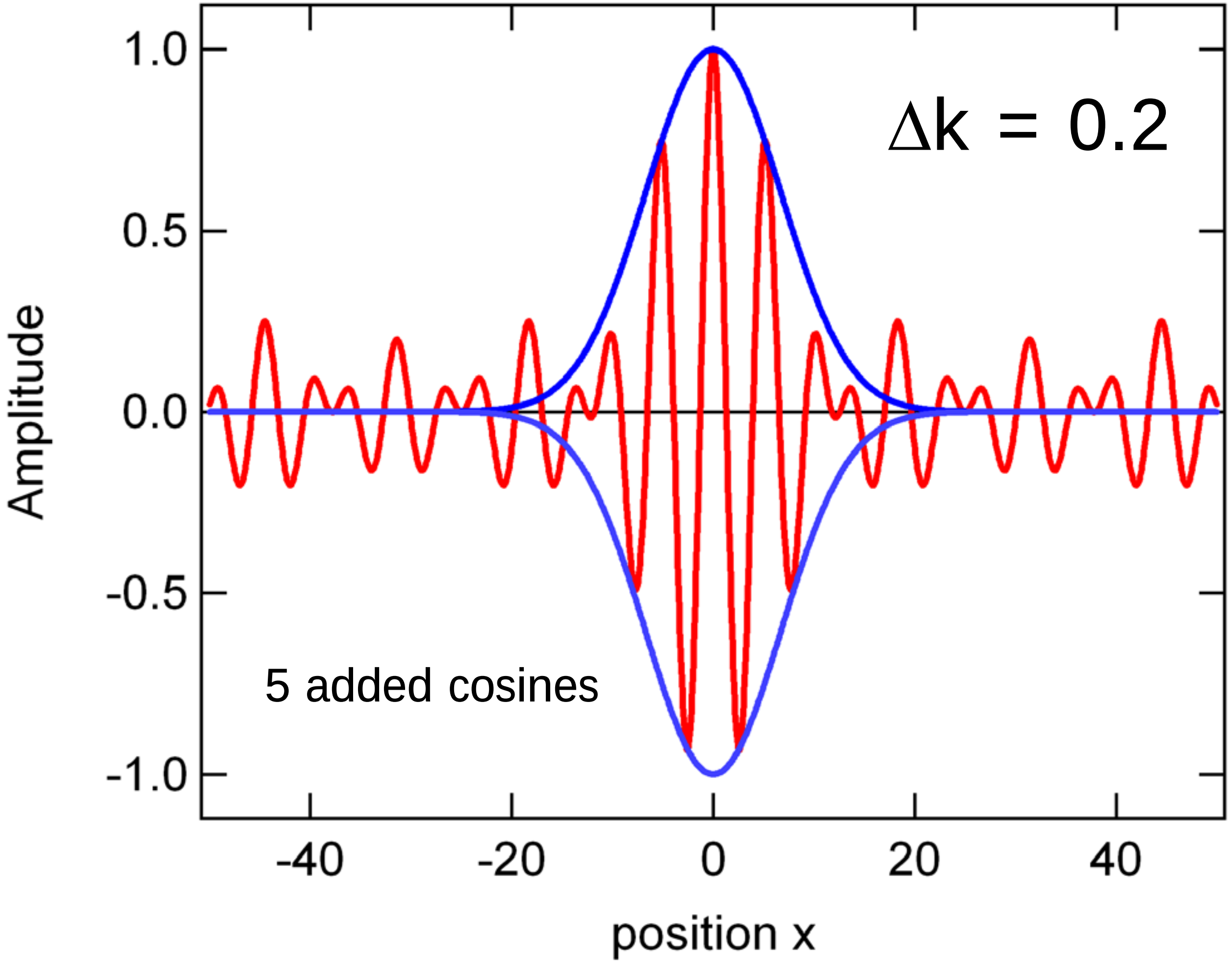
add more frequencies to make bandwidth Δk

$$\Psi(\mathrm{x}) = \sin(0.9kx) + \sin(kx) + \sin(1.1kx)$$

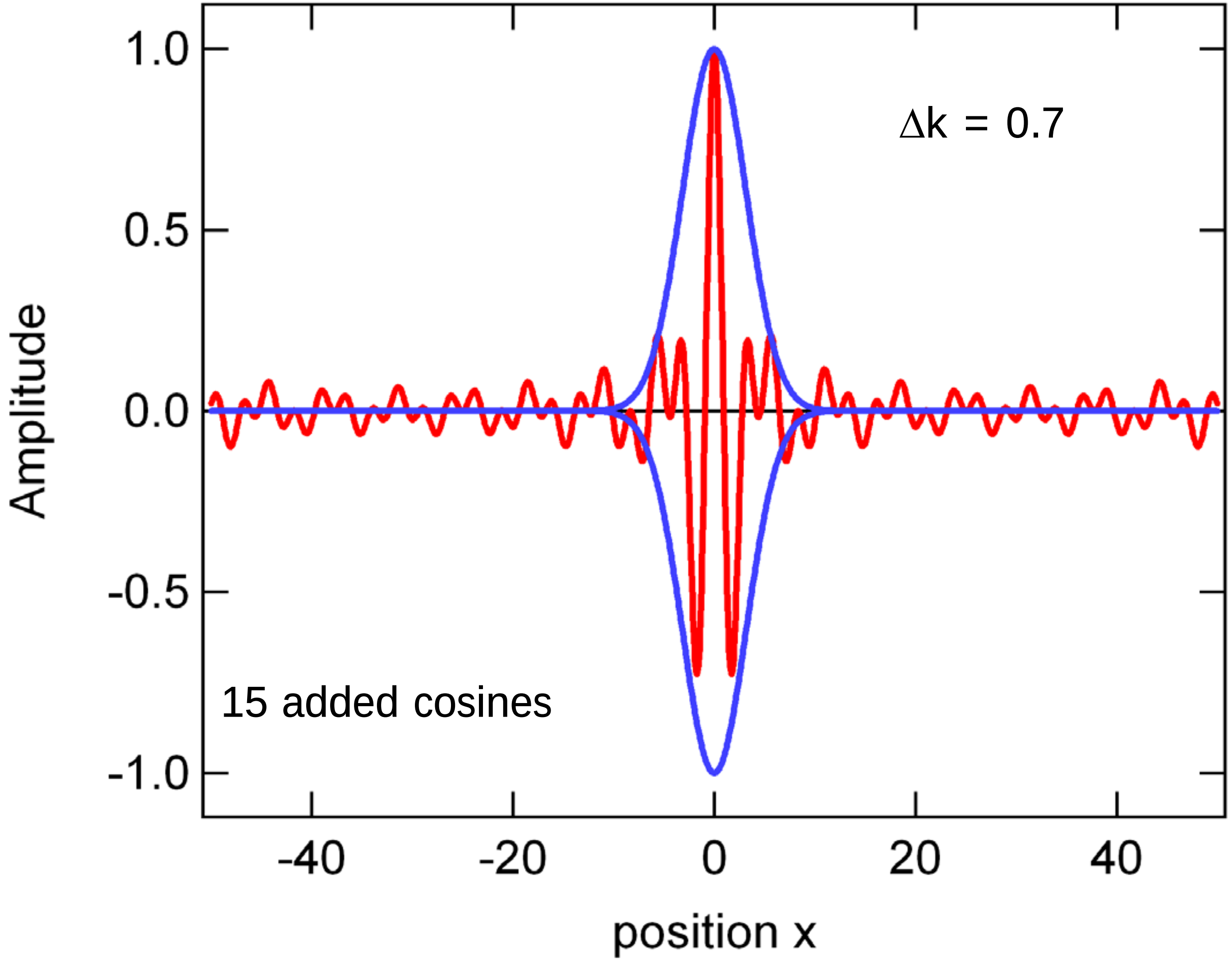


As the Δk bandwidth increases the position in Δx-space becomes more defined

$$\Psi(\mathrm{x}) = \sin(0.8kx) + \sin(0.9kx) + \sin(kx) + \sin(1.1kx) + \sin(1.2kx)$$



As the Δk bandwidth increases the position in Δx-space becomes more defined



Fourier transform related pairs

Position and momentum are related by a Fourier transform.

$$x \longleftrightarrow p$$

$$\Delta x \Delta p \geq \hbar/2$$

Time and energy are related by a Fourier transform.

$$t \longleftrightarrow E$$

There is an uncertainty relationship for both of these related pairs.

Thus, for time and energy we have

$$\Delta t \Delta E \geq \hbar/2$$

Gaussian Functions

A Gaussian function has the form $\exp\{ -\alpha(x - x_0)^2 \}$. The Gaussian indicated is centered about the point x_0. The Fourier transform of a Gaussian in x-space is a Gaussian in k-space. Since $p = \hbar k$ we also call this momentum space. The figure shows the inverse relationship.