The probability interpretation

The wave function is related to the probability for finding a particle in a given region of space. The relationship is given by:

$$P = \int \Psi^2 dV$$

If we integrate the square of the wave function over a given volume we find the probability that the particle is in that volume. In order for this to be true the integral over all space must be one.

$$1 = \int_{all\ space} \Psi^2 dV$$

If this equation holds then we say that the wave function is normalized.

The normalized bound state wave function

For the wave function we have been considering, “all space” is the region from 0 to L. So the normalization constant A can be determined from the integral:

$$1 = \int_0^L \Psi^2 dx = \int_0^L A^2 \sin\left(\frac{n\pi x}{L}\right)^2 dx = A^2 \int_0^L \sin\left(\frac{n\pi x}{L}\right)^2 dx$$

The solution is L/2. Thus, we have:

$$1 = A^2 \frac{L}{2}\ ,\ A^2 = \frac{2}{L}\ ,\ A = \sqrt{\frac{2}{L}}$$

When we multiply the wave function by this value we say that it is normalized.

The probability of finding the particle in a given region of space

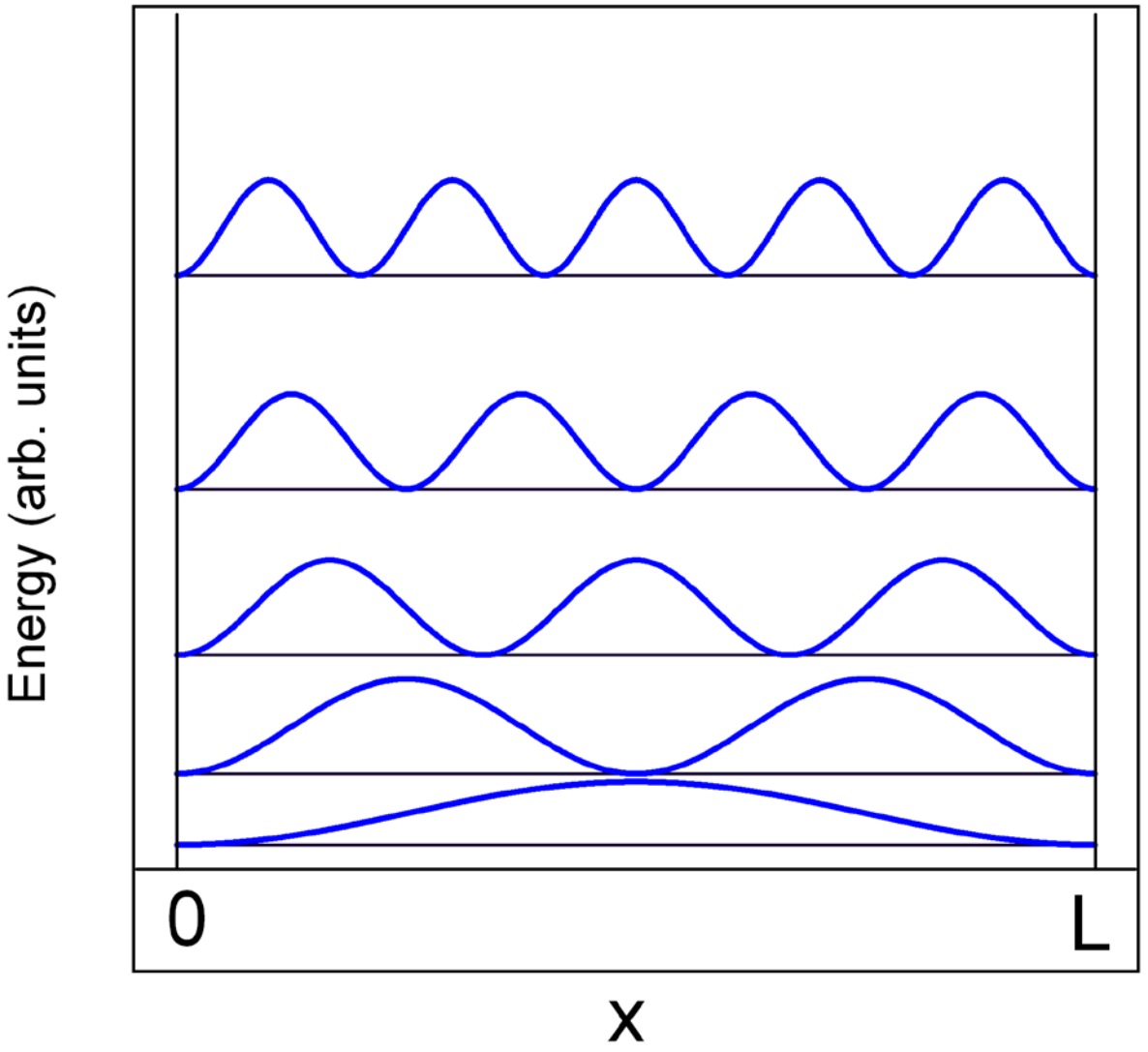
Using the normalized wave function

$$\Psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

one can calculate the probability of finding the particle in any region of space. Since the wave function is normalized, the probability P is a number between 0 and 1. For example: What is the probability that the particle is between 0.2L and 0.4L?

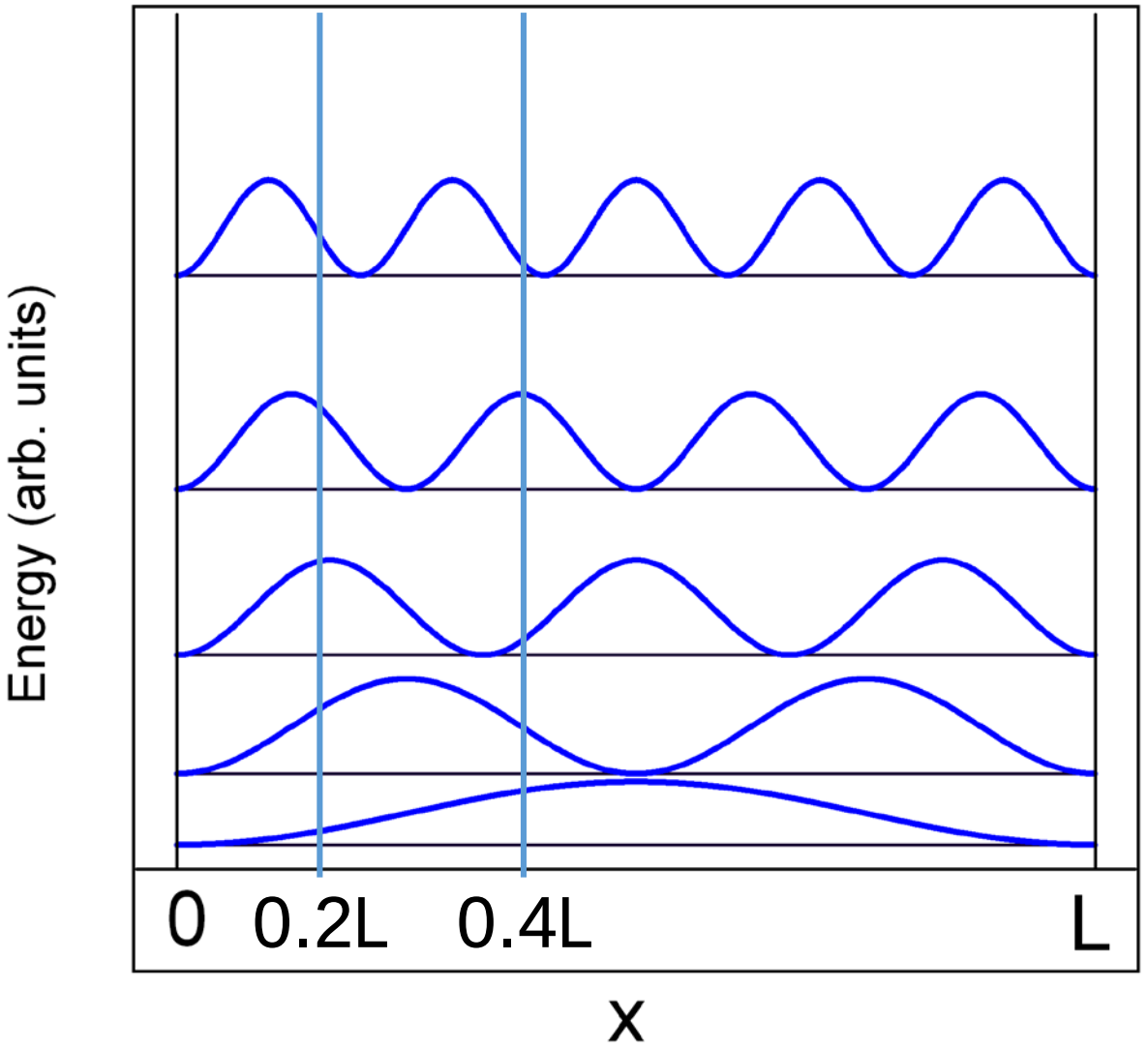
$$P = \int_{0.2L}^{0.4L} \Psi(x)^2 dx = \frac{2}{L} \int_{0.2L}^{0.4L} \sin\left(\frac{n\pi x}{L}\right)^2 dx \approx 0.25$$

The appearance of the probability Ψ^2



The square of the wave function is positive in all regions of space. Shown on the left are the squared solutions for the first five energy levels of the particle in a box. Using integration we have shown that the area underneath the lowest one is 2/L.

The appearance of the probability Ψ^2



On this diagram we have drawn two lines that represent the limits for our calculation of probability. These lines are at 0.2L and 0.4L. Our calculation shows that the probability of the particle being in this region of the box is P ~ 0.25. The tilde means that this approximate. We are not striving for high accuracy since this a demo calculation.