Preparation of the spin echo

Now that we understand the fundamentals of the product operator, we apply this idea to the spin echo:

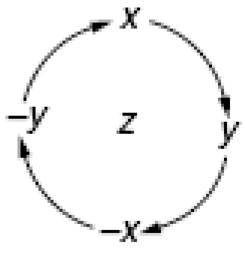
<u>Spin echo</u>

$$90°\,(x) \overset{a}{\text{—}} \text{delay } \tau \overset{b}{\text{—}} 180°\,(x) \overset{e}{\text{—}} \text{delay } \tau \overset{f}{\text{—}} \text{acquire}$$

First, we prepare with a π/2 pulse:

$$I_z \xrightarrow{\omega_1 t_p I_x} -I_y \qquad (a)$$

Then the spins evolve in the x,y plane: $H_{free} = \Omega I_z$



$$-I_y \xrightarrow{\Omega\tau I_z} -\cos\Omega\tau I_y + \sin\Omega\tau I_x \qquad (b)$$

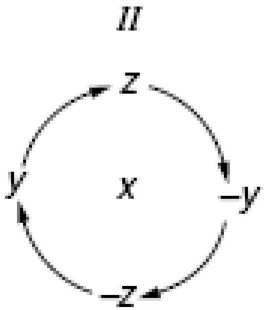
Application of the π-pulse

90° (x) $\overset{a}{—}$ delay τ $\overset{b}{—}$ 180° (x) $\overset{e}{—}$ delay τ $\overset{f}{—}$ acquire

Apply the π-pulse $\qquad H_{pulse,x} = \omega_1 I_x$

$$-\cos \Omega\tau I_y + \sin \Omega\tau I_x \xrightarrow{\omega_1 t_p I_x} \sigma(e)$$

$$-\cos \Omega\tau I_y \xrightarrow{\omega_1 t_p I_x} -\cos \Omega\tau \cos\omega_1 t_p I_y - \cos \Omega\tau \sin \omega_1 t_p I_z$$



Since the flip angle here is $\omega_1 t_p = \pi$ the second term on the right goes to zero and the first term changes sign (cos π = -1). Such that:

$$-\cos \Omega\tau I_y \xrightarrow{\pi I_x} \cos \Omega\tau I_y$$

Treatment of I_y and I_x terms

90° (x) $\overset{a}{\text{—}}$ delay τ $\overset{b}{\text{—}}$ 180° (x) $\overset{e}{\text{—}}$ delay τ $\overset{f}{\text{—}}$ acquire

$$-\cos \Omega\tau I_y + \sin \Omega\tau I_x \xrightarrow{\omega_1 t_p I_x} \sigma(e)$$

$$-\cos \Omega\tau I_y \xrightarrow{\pi I_x} \cos \Omega\tau I_y$$

The second term ($\sin\Omega\tau\ I_x$) on the left is unchanged because it is not affected by a rotation about x. Therefore after the 180° pulse the following is obtained:

$$-\cos \Omega\tau I_y + \sin \Omega\tau I_x \xrightarrow{\pi I_x} \cos \Omega\tau I_y + \sin \Omega\tau I_x$$

Evolution following the π-pulse

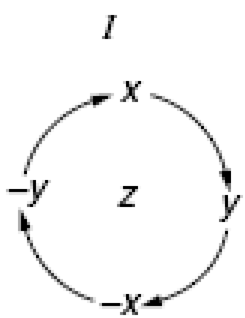
$$90° \text{ (x)} \overset{a}{\text{—}} \text{delay } \tau \overset{b}{\text{—}} 180° \text{ (x)} \overset{e}{\text{—}} \text{delay } \tau \overset{f}{\text{—}} \text{acquire}$$

$$-\cos \Omega\tau I_y + \sin \Omega\tau I_x \xrightarrow{\pi I_x} \cos \Omega\tau I_y + \sin \Omega\tau I_x$$

Evolution (e) involves separate treatment of each term on the right:

$$\cos \Omega\tau I_y \xrightarrow{\Omega\tau I_z} \cos \Omega\tau \cos \Omega\tau I_y - \sin \Omega\tau \cos \Omega\tau I_x$$

$$H_{free} = \Omega I_z$$



$$\sin \Omega\tau I_x \xrightarrow{\Omega\tau I_z} \cos \Omega\tau \sin \Omega\tau I_x + \sin \Omega\tau \sin \Omega\tau I_y$$

Refocusing of the spins

$$90° \text{ (x)} \overset{a}{—} \text{delay } \tau \overset{b}{—} 180° \text{ (x)} \overset{e}{—} \text{delay } \tau \overset{f}{—} \text{acquire}$$

Collecting the terms in I_x and I_y gives us:

$$(\cos \Omega\tau \cos \Omega\tau + \sin \Omega\tau \sin \Omega\tau) I_y +$$

$$(\cos \Omega\tau \sin \Omega\tau - \sin \Omega\tau \cos \Omega\tau) I_x$$

The terms multiplying I_x goes to 0 and the terms multiplying I_y goes to 1 because of the identity: $\cos^2\theta + \sin^2\theta = 1$

Therefore:

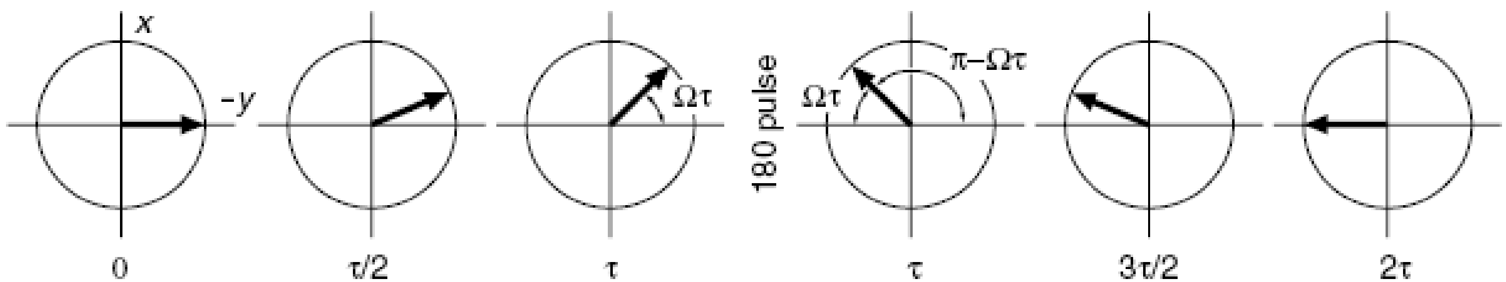
$$I_z \xrightarrow{90°(x)-\tau-180°-(x)-\tau-} I_y$$

Product operators vs. vector

Spin echo

90° (x) $\overset{a}{—}$ delay τ $\overset{b}{—}$ 180° (x) $\overset{e}{—}$ delay τ $\overset{f}{—}$ acquire

Vector



Product operator

$$I_z \xrightarrow{90°(x)\text{-}\tau\text{-}180°\text{-}(x)\text{-}\ \tau\text{-}} I_y$$