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This problem has two parts. In part 1 we need to solve for the wavelength of the transition.

We have learned that the Rydberg equation has the form:

$$\tilde{\nu} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

and we can convert to wavelength using the relation:

$$\tilde{\nu} = \frac{10^7}{\lambda}$$

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Putting these together we can write the Rydberg formula as:

$$\frac{10^7}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

And finally we can express it in a useful form for wavelengths (in nm):

$$\frac{1}{\lambda} = \frac{R}{10^7}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

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Finally we can solve for wavelength:

$$\lambda = \frac{10^7}{R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)} = \frac{10^7}{109{,}700\left(\frac{1}{1} - \frac{1}{16}\right)} = 97.2\ nm$$

Now we consider part 2 of the problem. We need to determine the momentum of the electron and then the recoil momentum of the nucleus (proton). conservation of momentum tells us that $m_e v_e = m_p v_p$

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We can use the DeBroglie relation to calculate the energy of the electron.

$$p = \frac{h}{\lambda} = \frac{(6.626\ x\ 10^{-34}\ Js)}{97.2\ x\ 10^{-9}\ m} = 6.816\ x\ 10^{-27}\ kg\ m/s$$

This value seems really small until we think about how small the mass is. We do not need to solve for the electron speed since all that is needed is the recoil of the nucleus.

$$v_p = \frac{p}{m_p} = \frac{6.816\ x\ 10^{-27}\ kg\ m/s}{1.672\ x\ 10^{-27}\ kg} = 4.1\ m/s$$