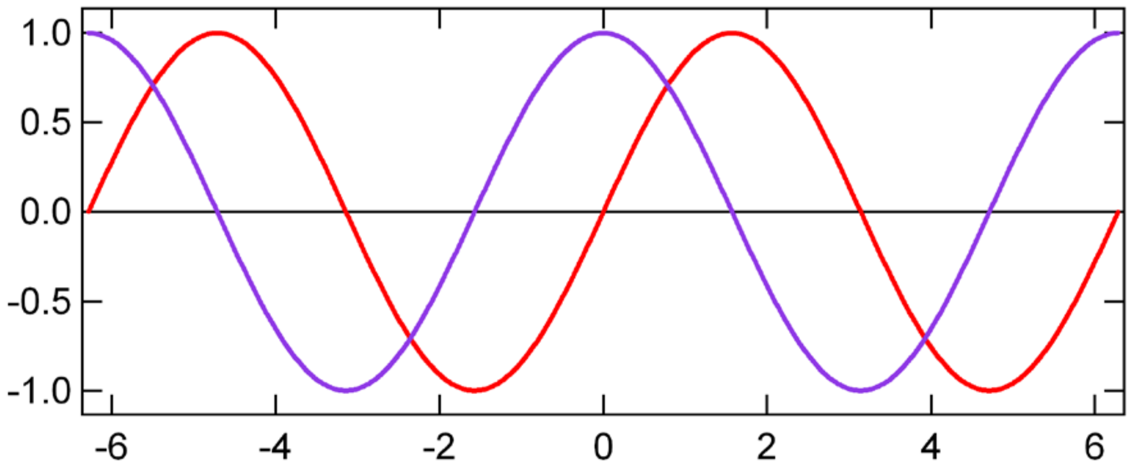
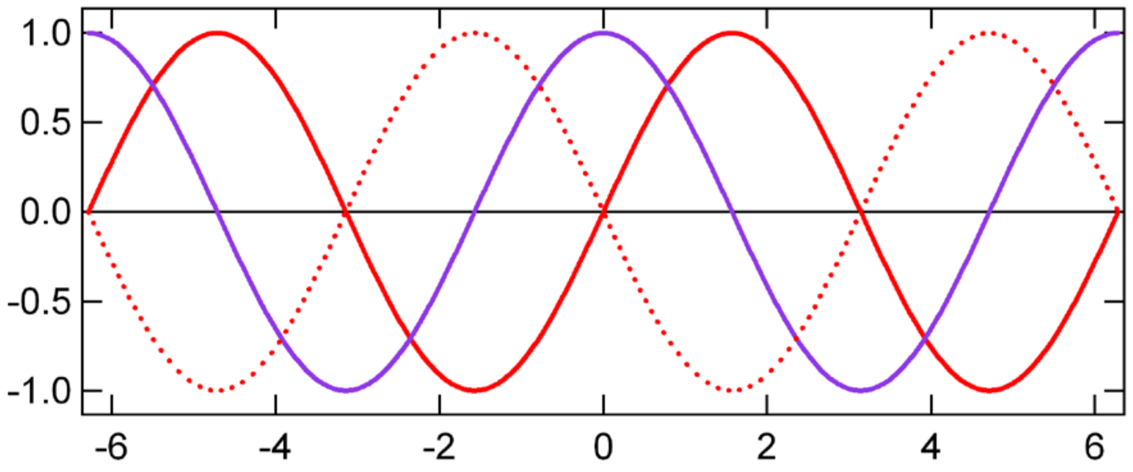
The derivative of sin(x)

$$\frac{d}{dx} \sin(x) = \cos(x)$$



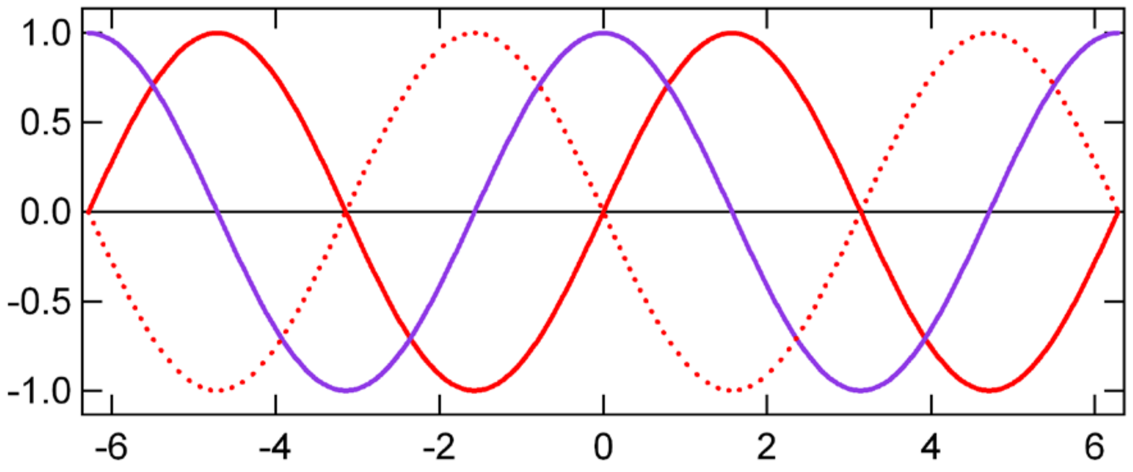
The derivative of cos(x)

$$\frac{d}{dx} \cos(x) = -\sin(x)$$



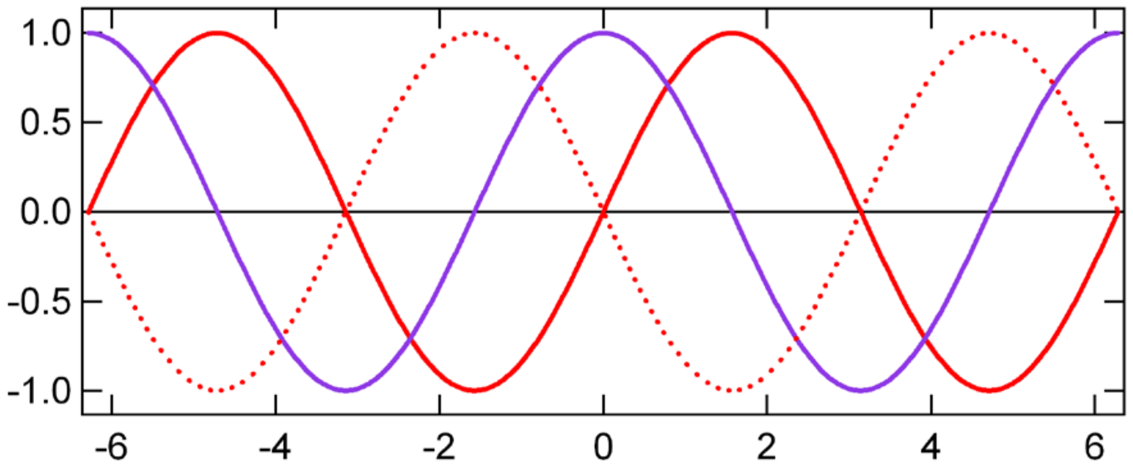
The second derivative of sin(x)

$$\frac{d}{dx}\left(\frac{d}{dx}\sin(x)\right) = -\sin(x)$$



The second derivative of sin(x)

$$\frac{d^2}{dx^2} \sin(x) = -\sin(x)$$



Sin(x) is an eigenfunction

If we define $-\dfrac{d^2}{dx^2}$ as an operator G then we have:

$$-\frac{d^2}{dx^2}\sin(x) = \sin(x)$$

which can be written as:

$$G\sin(x) = \sin(x)$$

This is a simple example of an operator equation that is closely related to the Schrödinger equation.

Sin(kx) is also an eigenfunction

We can make the problem more general by including a constant k. This constant is called a wavevector. It determines the period of the sin function. Now we must take the derivative of the sin function and also the function kx inside the parentheses (chain rule).

$$-\frac{d}{dx}\sin(kx) = -k\cos(kx)$$

$$-\frac{d^2}{dx^2}\sin(kx) = k^2\sin(kx)$$

Here we call the value k^2 the eigenvalue.

Sin(kx) is an eigenfunction of the Schrödinger equation

The example we are using here can easily be expressed as the Schrodinger equation for wave in space. We only have to add a constant.

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\sin(kx) = \frac{\hbar^2 k^2}{2m}\sin(kx)$$

In this equation $\hbar$ is Planck's constant divided by 2π and m is the mass of the particle that is traveling through space. The eigenfunction is still sin(kx), but the eigenvalue in this equation is actually the energy.

The Schrödinger equation

Based on these considerations we can write a compact form for the Schrödinger equation.

$$H\Psi = E\Psi$$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

Energy operator, Hamiltonian

$$E = \frac{\hbar^2 k^2}{2m}$$

Energy eigenvalue, Energy

$$\Psi = \sin(kx)$$

Wavefunsction

The definition of momentum

The momentum is related to the kinetic energy. Classically
The kinetic energy is:

$$E = \frac{1}{2} m v^2$$

The momentum is:

$$p = mv$$

So the classical relationship is:

$$E = \frac{p^2}{2m}$$

If we compare this to the quantum mechanical energy:

$$E = \frac{\hbar^2 k^2}{2m}$$

we see that: $p = \hbar k$