A. Calculate a realistic estimate for the temperature at the surface of Mercury using the black body radiation formula. Use the following facts.

Radius of Mercury: 2440 km.

Distance of Mercury from the sun: 5,820,000 km.

Note: Mercury does not rotate on its axis. Please calculate the temperature on the sunny side! You may assume that the dark side is very cold (i.e. near T = 0 K).

Solution: $R_{merc} = 2440 \times 10^6$ m

$R_{to\_sun} = 5.82 \times 10^{10}$ m

Given the radiant power of the sun at its surface is 3.2×10^{26} W as we calculated in class, the insolation on Mercury is given by this power divided by the area of a sphere at the radius equal to the distance from Mars to the sun.

The area is $A = 4\pi R_{to\_sun}^2 = 4(3.14159)(5.82 \times 10^{10}\ \text{m})^2 = 4.26 \times 10^{22}\ \text{m}^2$.

The insolation is $I = 3.9 \times 10^{26}\ \text{W}/4.26 \times 10^{22}\ \text{m}^2 = 9165\ \text{W/m}^2$.

It is about seven times as much flux as we receive here on earth.

Now we calculate the total power that Mercury absorbs. For this we need the cross sectional area of Mercury.

$A_{mars\_cs} = \pi R_{merc}^2 = 3.14159(2.44 \times 10^6\ \text{m})^2 = 1.87 \times 10^{13}\ \text{m}^2$.

Thus the total power is:

$P_{abs} = IA_{mars\_cs} = (9165\ \text{W/m}^2)(1.87 \times 10^{13}\ \text{m}^2) = 1.70 \times 10^{17}$ W.

The corresponding value for the earth is 6×10^{17} W or about 4 times larger.

Now, we must reason that if Mars is in equilibrium the power emitted as black body radiation must equal the power absorbed.

$P_{emit} = P_{abs}$

And

$P_{emit} = \sigma T_{merc}^4 A_{merc} = P_{abs}$

Since Mercury does not rotate the effective emitting area is the half sphere on the sunny side:

$A_{merc} = 4\pi R_{merc}^2/2 = 2(3.14159)(2.44 \times 10^6\ \text{m})^2 = 3.74 \times 10^{13}\ \text{m}^2$.

$$T_{merc} = (P_{abs} / A_{merc}/\sigma)^{1/4}$$
$$= (1.7 \times 10^{17}\ \text{W}/3.74 \times 10^{13}\ \text{m}^2/5.67 \times 10^{-8}\ \text{kg s}^{-3}\ \text{K}^{-4})^{1/4}$$
$$= 532\ \text{K}.$$

Temperature = ______532 K____________.

B. What wavelength is the peak of the black body emission from Neptune?

Solution:

$\lambda_{max} T = 2.897 \times 10^6$ nm-K

$\lambda_{max} = 2.897 \times 10^6\ \text{nm-K}/T = 2.897 \times 10^6\ \text{nm-K}/532\ \text{K} = 5440\ \text{nm} = 5.44\ \mu$

Wavelength = _______5.44 microns ______________