What is the radiant power of the sun?

First calculate the area and then the flux (power).

The area is $A = 4\pi R^2$

$A = 4(3.1416)(7 \times 10^8)^2$

$A = 6.16 \times 10^{18}\ m^2$

The flux is

$W = \sigma\ T^4$

$W = 5.6704 \times 10^{-8}\ (5780)^4$

$W = 6.33 \times 10^7$ Watts/m^2

The total power is

$P = WA = (5.19 \times 10^7$ Watts/$m^2)(6.16 \times 10^{18}\ m^2)$

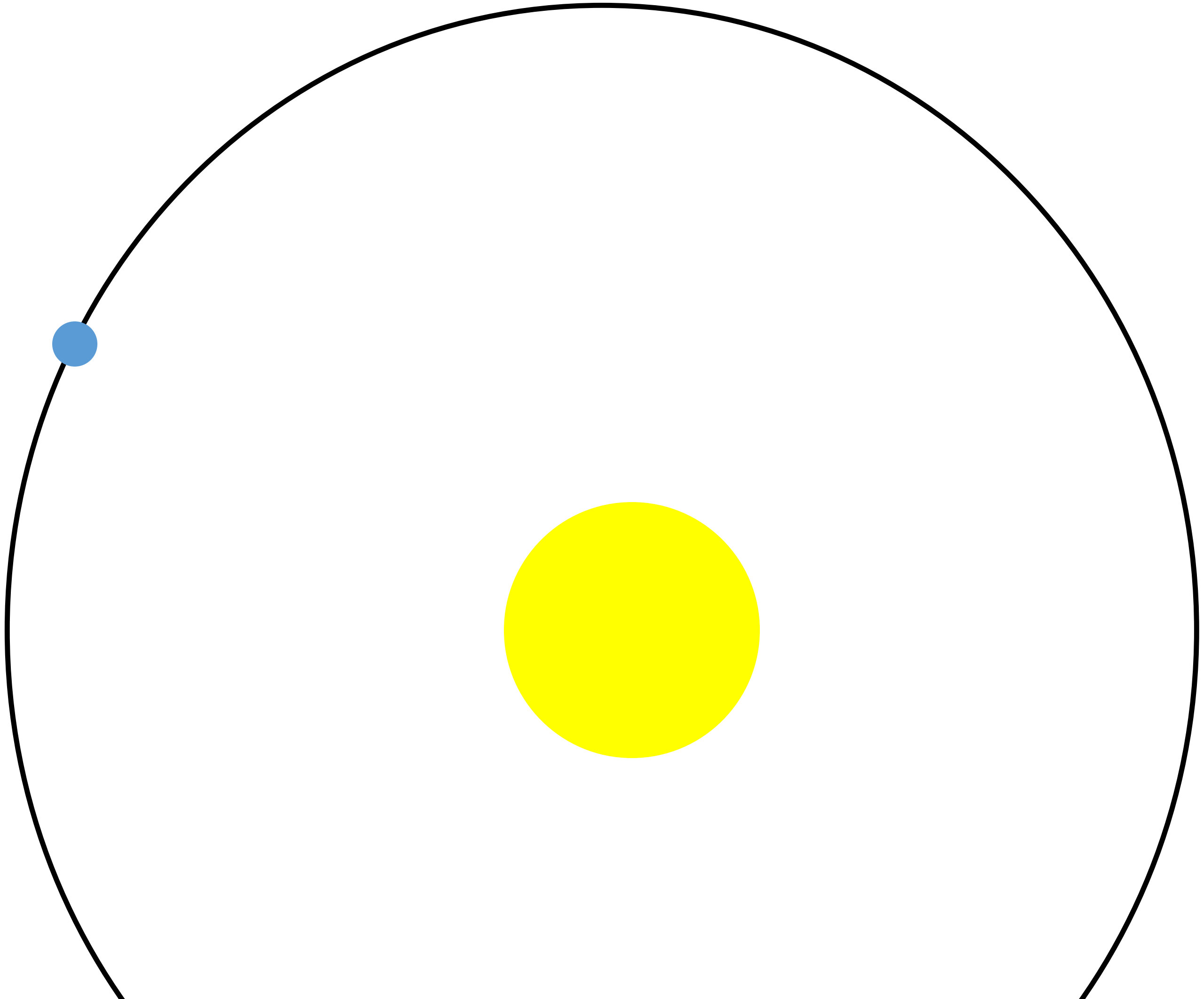
$P = 3.9 \times 10^{26}$ Watts

What is the radiant power at the surface of the earth?

We use the distance from the earth to the sun to obtain the flux at the earth.

The earth is
$R_e = 1.5 \times 10^8$ km
from the sun.

The area irradiated is
$A_e = 4\pi R_e^2$
$A_e = 2.83 \times 10^{23}\ m^2$



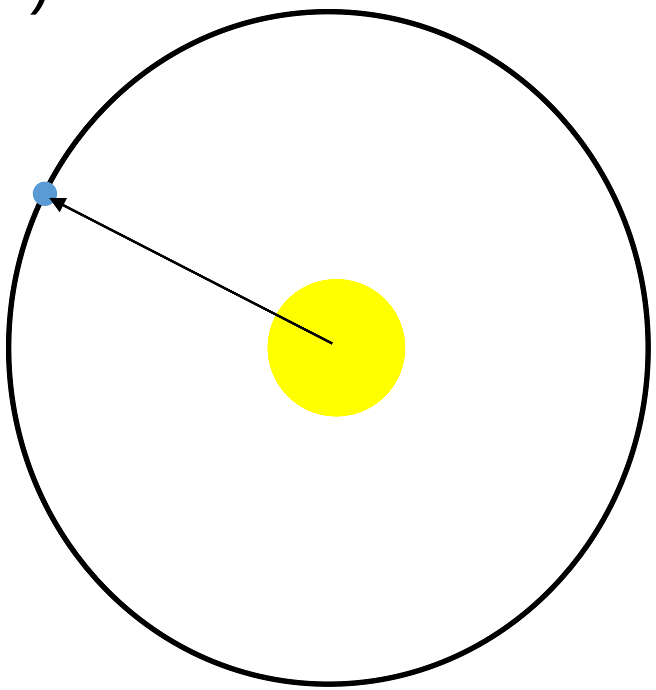
Calculation of the solar constant

The flux in space above the earth is called the insolation.
The insolation is the power coming from the sun divided by the total area at the radius of the earth.

$W_e = P/A_e = (3.9 \times 10^{26}\ \text{Watts})/(2.83 \times 10^{23}\ \text{m}^2)$

$W_e = 1.37 \times 10^3\ \text{Watts/m}^2$

This is very close to the measured value for radiation in space above the earth.



How much energy does the earth absorb?

The earth has a cross-sectional area of

$A_c = \pi R_{earth}^2$

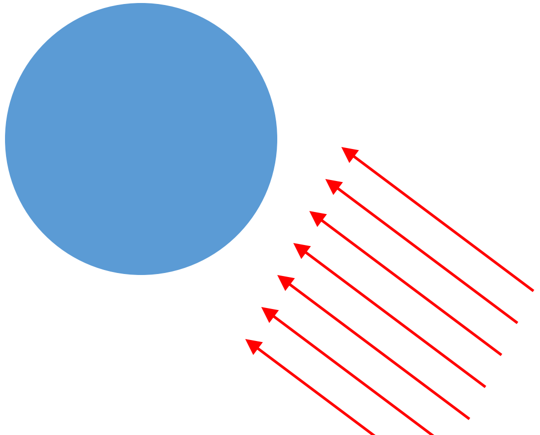
$A_c = (3.1416)(6.4 \times 10^6 \text{ m})^2$

$A_c = 1.3 \times 10^{14} \text{ m}^2$

$P_{abs} = W_e A_c$

$P_{abs} = (1.37 \times 10^3 \text{ Watts/m}^2)(1.3 \times 10^{14} \text{ m}^2)$

$P_{abs} = 1.8 \times 10^{17} \text{ Watts}$

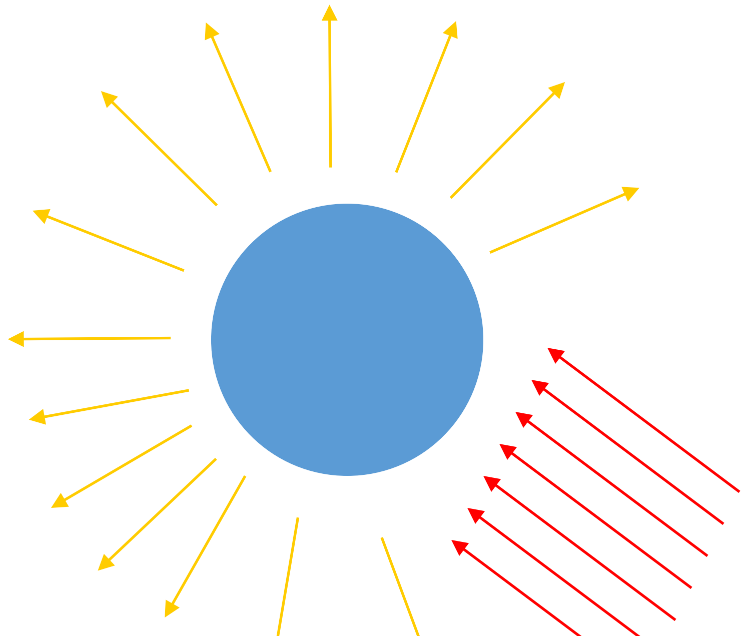


Using the equilibrium condition to calculate the earth's temperature

$P_{abs} = P_{emit} = 1.8 \times 10^{17}$ Watts
$P_{emit} = \sigma\, T_{earth}{}^4\, A_{earth}$ $[A_{earth} = 4\pi\, R_{earth}{}^2 = 5.1 \times 10^{14}\ m^2]$
$T_{earth} = (P_{emit}/\sigma\, A_{earth})^{1/4}$
$T_{earth} = (1.8 \times 10^{17}/5.6704 \times 10^{-8}/5.1 \times 10^{14})^{1/4}$
$T_{earth} = 279$ K

This is close, but it is a little cool.
Why is this?

What is 279 K in ^oC?
What is 279 K in ^oF?



One of the early goals of the thermal radiation theory was to calculate the temperature of the earth from first principles. We can do this using the Stefan-Boltzmann law provided we know the temperature of the sun, the radii of both the sun and the earth and distance between them.

First, we calculate the total radiant power of the sun using the established value of 5780 K for the temperature of the sun to calculate the flux W = 6.33×10^7 W/m^2. We can obtain an area of 6.16×10^{18} m^2 from the radius of 7×10^8 meters. Using the product of W and A we obtain a total power of 3.9×10^{26} watts. Of course, the earth only absorbs a miniscule fraction of this huge amount of energy.

To determine what fraction of the energy the earth absorbs we consider the fact that the energy propagates in a spherically expanding wave front as it leaves the sun. Thus, the energy per unit area (the flux) at the distance of the earth can be calculated by dividing the total power by the area of a sphere with a radius equal to the earth's distance from the sun. We call this flux the solar constant. Using a distance of 150,000,000 kilometers (1.5×10^{11} m) As the radius, we calculate that the sphere has an area of 2.83×10^{23} m^2. Hence, the solar constant is calculated to 1.37×10^3 W/m^2. This is in good agreement with the measured value of the incident flux at the top of the atmosphere. Of course, there are losses due to reflection by the atmosphere (albedo) and absorption of some of the radiation.

Our calculation ignores the atmosphere so we mention these factors only for sake of general interest. The earth is a target that absorbs the radiation based on its cross-sectional area. Using the radius of the earth as 6.4×10^6 m, that area is 1.3×10^{14} m^2. Thus, the total power absorbed by the earth is 1.8×10^{17} Watts. Finally we assume that the earth is at equilibrium so that this power is also the emitted power. Using this value for the power and the spherical area of the earth we can solve for the temperature of the earth, which is the only unknown. We find that the earth has an average temperature of 279 K. This value is a bit lower than the actual average value of 10 ^{o}C. The reason for the discrepancy is the atmosphere. There is a thin shell of gas that changes the surface temperature due to the greenhouse effect, albedo and other factors.