## Quantum Chemistry

## Lecture 7 Group Theory

Application to the LCAO Approach to MO Theory

NC State University

## Polyatomic molecules: $\mathrm{H}_{2} \mathrm{O}$

For polyatomic molecules appropriate atomic orbitals must be combined into symmetry adapted linear combinations (SALCs). We can identify two groups of atoms in $\mathrm{H}_{2} \mathrm{O}$. The central O atom (group 1) and the pair of H atoms (group 2).
$\mathrm{H}_{2} \mathrm{O}$ belongs to the $\mathrm{C}_{2 \mathrm{v}}$ point group. We have already seen that group 1 consists of the valence orbitals of the O atom:

$$
2 \mathrm{H}(1 \mathrm{~s})+\mathrm{O}(2 \mathrm{~s})+\mathrm{O}\left(2 \mathrm{p}_{\mathrm{x}}\right)+\mathrm{O}\left(2 \mathrm{p}_{\mathrm{y}}\right)+\mathrm{O}\left(2 \mathrm{p}_{z}\right)
$$

Since the oxygen is unmoved by every symmetry operation the character can be determined directly from the character table. However, the hydrogens need to be formed as SALCs first.

## Reducible rep for the H atoms

To treat the H atoms we look at the set of two H atoms, each having 1 s orbital. First we can consider the Reducible representation for the two 1 s orbitals.
$\mathrm{C}_{2 \mathrm{v}}$ character table and the reducible representation
for the $2 \mathrm{H}(1 \mathrm{~s})$ orbitals in water

| $\mathrm{C}_{2 \mathrm{v}}$ | E | $\mathrm{C}_{2}$ | $*_{\mathrm{v}}(\mathrm{xz})$ | $*_{\mathrm{v}}(\mathrm{yz})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 1 | 1 | 1 | 1 | z |
| $\mathrm{A}_{2}$ | 1 | 1 | -1 | -1 |  |
| $\mathrm{~B}_{1}$ | 1 | -1 | 1 | -1 | x |
| $\mathrm{B}_{2}$ | 1 | -1 | -1 | 1 | y |
| $*[\mathrm{H}(1 \mathrm{~s})]$ | 2 | 0 | 2 | 0 |  |



When we decompose this reducible representation we find:

$$
\begin{aligned}
\chi_{A_{1}} & =\frac{1}{4}((1)(2)(1)+(1)(0)(1)+(1)(2)(1)+(1)(0)(1))=1 \\
\chi_{A_{2}} & =\frac{1}{4}((1)(2)(1)+(1)(0)(1)+(-1)(2)(1)+(1)(0)(1))=0 \\
\chi_{B_{1}} & =\frac{1}{4}((1)(2)(1)+(-1)(0)(1)+(1)(2)(1)+(-1)(0)(1))=1 \\
\chi_{B_{2}} & =\frac{1}{4}((1)(2)(1)+(-1)(0)(1)+(-1)(2)(1)+(1)(0)(1))=0
\end{aligned}
$$

## Linear combinations of H atoms

We can use a method known as projection operators to Determine what the linear combinations of H atoms Correspond to in terms of symmetry adapted atomic orbitals.

These are easily visualized in this case:


$$
\chi_{\mathrm{A}_{1}}=1 \mathrm{~s}_{\mathrm{A}}+1 \mathrm{~s}_{\mathrm{B}}
$$



$$
\chi_{\mathbf{B}_{1}}=1 \mathrm{~s}_{\mathrm{A}}-1 \mathrm{~s}_{\mathrm{B}}
$$

## Method of Projection Operators

To use the projection operator method we choose one member of a symmetry related set of orbitals. We then apply the symmetry operations of the group to that orbital and observe how that AO is transformed into other orbitals in the molecule.


## Method of Projection Operators

We can see that $\mathrm{H}_{2}$ either remains in its location or is transformed into $\mathrm{H}_{3}$ by the symmetry operations. Thus, we find that a table would look like

| $\mathrm{C}_{2 v}$ | $E$ | $\mathrm{C}_{2}$ | $\sigma_{v}(x z)$ | $\sigma_{v}{ }^{\prime}(y z)$ |
| :--- | :--- | :--- | :--- | :--- |
| $P$ | $\mathrm{H}_{2}$ | $\mathrm{H}_{3}$ | $\mathrm{H}_{2}$ | $\mathrm{H}_{3}$ |
| $\mathrm{~A}_{1}$ | 1 | 1 | 1 | 1 |
| $\mathrm{~B}_{1}$ | 1 | -1 | 1 | -1 |

The row labeled $P$ shows the projections. Finally, we apply the characters of the irreps for that set to construct MOs. We determined that there are two MOs, which transform as $a_{1}$ and $b_{1}$. We multiply the projected AOs by the characters of these irreps to find the two linear combinations.

## Polyatomic molecules

The reducible representations for hydrogen SALCs are $a_{1}$ and $b_{1}$. The oxygen orbitals (read from the character Table are:
$2 \mathrm{~s}=\mathrm{a}_{1}$
$2 p_{z}=a_{1}$
$2 p_{x}=b_{1}$
$2 p_{y}=b_{2}$


Note that the energies are obtained from a calculation. However, the forms of the orbitals can be predicted based on symmetry considerations.

## HOMO to LUMO transition in $\mathrm{H}_{2} \mathrm{O}$

top

side


| Eigenstate | Irrep | Hartrees | eV | upation |
| :---: | :---: | :---: | :---: | :---: |
| + 1 | a1.1 | -18.748477 | -510.1 | 1.000 |
| 2-1 | a1.1 | -18.748477 | -510.1 | 1.000 |
| $3+2$ | a1.1 | -0.917545 | -24.968 | 1.000 |
| 4-2 | a1.1 | -0.917545 | -24.968 | 1.000 |
| $5+1$ | b1.1 | -0.471788 | -12.838 | 1.000 |
| 6-1 | b1.1 | -0.471788 | -12.838 | 1.000 |
| $7+3$ | a1. 1 | -0.333240 | -9.068 | 1.000 |
| 8-3 | a1.1 | -0.333240 | -9.068 | 1.000 |
| $9+1$ | b2.1 | -0.253144 | -6.888 | 1.000 |
| 10-1 | b2.1 | -0.253144 | -6.888 | 1.000 |
| $11+4$ | a1.1 | 0.023886 | 0.650 | 0.000 |
| 12-4 | a1.1 | 0.023886 | 0.650 | 0.000 |
| $13+2$ | b1.1 | 0.093912 | 2.555 | 0.000 |
| 14-2 | b1.1 | 0.093912 | 2.555 | 0.000 |
| $19+2$ | b2.1 | 0.563271 | 15.327 | 0.000 |
| 20-2 | b2.1 | 0.563271 | 15.327 | 0.000 |
| 27-1 | a2.1 | 0.926496 | 25.211 | 0.000 |
| $28+1$ | a2.1 | 0.926496 | 25.211 | 0.000 |

## Ethene

Molecules with p-systems are particularly important in Organic chemistry. The simplest of these is ethene.


Ethene belongs to the $\mathrm{D}_{2 h}$ point group. The carbons are symmetrically equivalent. The orbital set on each consists of

$$
C(2 s)+C\left(2 p_{x}\right)+C\left(2 p_{y}\right)+C\left(2 p_{z}\right)
$$

## Ethylene

The atomic orbitals are shown below:







The reducible representations of the symmetrically equivalent sets formed for the valence ao's of ethylene can be determined and decomposed in the following groups.

| $\mathrm{D}_{2 \mathrm{~h}}$ | E | Z | Y | X | i | XY | XZ | YZ | Irreps |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{H}(1 \mathrm{~s})$ | 4 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | $\mathrm{a}_{\mathrm{g}}+\mathrm{b}_{1 \mathrm{~g}}+\mathrm{b}_{2 \mathrm{u}}+\mathrm{b}_{3 \mathrm{u}}$ |
| $\mathrm{C}(2 \mathrm{~s})$ | 2 | 0 | 0 | 2 | 0 | 2 | 2 | 0 | $\mathrm{a}_{\mathrm{g}}+\mathrm{b}_{3 \mathrm{u}}$ |
| $\mathrm{C}\left(2 \mathrm{p}_{\mathrm{z}}\right)$ | 2 | 0 | 0 | -2 | 0 | -2 | 2 | 0 | $\mathrm{~b}_{2 \mathrm{~g}}+\mathrm{b}_{1 \mathrm{u}}$ |
| $\mathrm{C}\left(2 \mathrm{p}_{\mathrm{x}}\right)$ | 2 | 0 | 0 | 2 | 0 | 2 | 2 | 0 | $\mathrm{a}_{\mathrm{g}}+\mathrm{b}_{3 \mathrm{u}}$ |
| $\mathrm{C}\left(2 \mathrm{p}_{\mathrm{v}}\right)$ | 2 | 0 | 0 | -2 | 0 | 2 | -2 | 0 | $\mathrm{~b}_{1 \mathrm{~g}}+\mathrm{b}_{2 \mathrm{u}}$ |

## Ethylene

Since the members of all of the sets can be exchanged completely with only the rotational operations, only the $\mathrm{C}_{2}$ characters of the above character table need be considered

| $\mathrm{D}_{2 \mathrm{~h}}$ | E | Y | Z | X |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{a}_{\mathrm{g}}$ | +1 | +1 | +1 | +1 |
| $\mathrm{~b}_{1 \mathrm{~g}}$ | +1 | -1 | +1 | -1 |
| $\mathrm{~b}_{2 \mathrm{~g}}$ | +1 | +1 | -1 | -1 |
| $\mathrm{~b}_{1 \mathrm{u}}$ | +1 | -1 | +1 | -1 |
| $\mathrm{~b}_{2 u}$ | +1 | +1 | -1 | -1 |
| $\mathrm{~b}_{3 \mathrm{u}}$ | +1 | -1 | -1 | +1 |

The set $\mathrm{E}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}$ correspond to the H atoms in ethene. We can use the table above to construct the SALCs for the irreps that were determined.

## Ethene

## The normalized SALCs can now be constructed:



## Ethene

The normalized SALCs can now be constructed:


| 1 | $a_{g}$ | -268.881 | 2 |
| :--- | :--- | :--- | :--- |
| 2 | $b_{1 u}$ | -268.863 | 2 |
| 3 | $a_{g}$ | -18.668 | 2 |
| 4 | $b_{1 u}$ | -14.148 | 2 |
| 5 | $b_{2 u}$ | -11.376 | 2 |
| 6 | $a_{g}$ | -10.115 | 2 |
| 7 | $b_{3 g}$ | -8.464 | 2 |
| 8 | $b_{3 u}$ | -6.613 | 2 |
| 9 | $b_{2 g}$ | -0.817 | 0 |
| 10 | $a_{g}$ | 2.541 | 0 |
| 11 | $b_{2 u}$ | 2.612 | 0 |
| 12 | $b_{1 u}$ | 3.337 | 0 |
| 13 | $b_{3 g}$ | 5.499 | 0 |
| 14 | $b_{2 g}$ | 13.552 | 0 |

## Transition moment for ethene

Note that the issue with this calculation is that the coordinate system in the quantum chemistry program is not the same as the one in the textbook example.

Question: which polarization of electromagnetic radiation can excite the $b_{3 g}->b_{2 u}$ transition?

Answer: An allowed transition must have transition moment that contains the totally symmetric representation. In the $D_{2 h}$ point group this means that the direct product

$$
\Gamma_{\mathrm{g}} \Gamma_{\mathrm{EM}} \Gamma_{\mathrm{e}}=\mathrm{A}_{\mathrm{lg}}
$$

One way to find this is to calculate the direct product for each basis vector that contains $\mathrm{x}, \mathrm{y}$ or z .

## MO diagram for the HOMO and LUMO



## Polarization of the allowed transition

Rather than try three different direct products, it is effective to note that the $A_{g}$ product will be achieved if

$$
\Gamma_{g} \Gamma_{e}=\Gamma_{x} \text { or } \Gamma_{y} \text { or } \Gamma_{z}
$$

In this case, we can calculate the direct product from the known HOMO and LUMO irreducible representations.
The direct product of $B_{3 u}$ and $B_{2 g}$ is

| $\mathbf{D}_{\mathbf{2 h}}$ | E | $\mathrm{C}_{2}$ | $\mathrm{C}_{2}{ }^{\prime}$ | $\mathrm{C}_{2}{ }^{\prime \prime}$ | $i$ | $\sigma_{\mathrm{v}}$ | $\sigma_{\mathrm{v}}{ }^{\prime}$ | $\sigma_{\mathrm{v}}{ }^{\prime \prime}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{B}_{3 \mathrm{ul}}$ | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| $\mathrm{~B}_{2 g}$ | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| $\Gamma_{\text {trans }}$ | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |

We can see by inspection that $\Gamma_{\text {trans }}=\mathrm{B}_{1 \mathrm{u}}$.

| $\mathbf{D}_{\mathbf{2 h}}$ | E | $\mathrm{C}_{2}$ | $\mathrm{C}_{2}{ }^{\prime}$ | $\mathrm{C}_{2}{ }^{\prime}$ | $i$ | $\sigma_{v}$ | $\sigma_{v}{ }^{\prime}$ | $\sigma_{v}{ }^{\prime}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{\mathrm{g}}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\mathrm{~B}_{\mathrm{g}}$ | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| $\mathrm{~B}_{2 \mathrm{~g}}$ | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| $\mathrm{~B}_{3 \mathrm{~g}}$ | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| $\mathrm{~A}_{u}$ | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| $\mathrm{~B}_{\mathrm{u}}$ | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| $\mathrm{~B}_{2 u}$ | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| $\mathrm{~B}_{3 u}$ | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |

The character table tells us that that $\mathrm{B}_{1 \mathrm{u}}$ corresponds to a polarization along $x$ (where $x$ is the internuclear bond vector).

## Formaldehyde



In the case of formaldehyde there is one additional atom beyond the water example in $\mathrm{C}_{2 v}$. Therefore, we Now have three symmetry related sets. The O, C and H atoms. However, the symmetry of the C and O atoms are the same since they both lie along the symmetry axis. Thus, we can construct three different Sets of symmetry adapted atoms orbitals and then combine them.

Formaldehyde


We can that the HOMO $\left(b_{1}\right)$ to LUMO $\left(b_{2}\right)$ Transition is forbidden.

| $\mathrm{C}_{2 \mathrm{v}}$ | E | $\mathrm{C}_{2}$ | $\sigma_{\mathrm{v}(\mathrm{Xz)}}$ | $\sigma_{\mathrm{v}(\mathrm{yz})}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{B}_{1}$ | 1 | -1 | 1 | -1 |
| $\mathrm{~B}_{2}$ | 1 | -1 | -1 | 1 |
| $\mathrm{~A}_{2}$ | 1 | 1 | -1 | -1 |

MO Irrep eV Occupation

| 1 | $a_{1}$ | -510.777 | 2.000 |
| :---: | :---: | :---: | :---: |
| 2 | $a_{1}$ | -271.620 | 2.000 |
| 3 | $a_{1}$ | -26.478 | 2.000 |
| 4 | $a_{1}$ | -15.675 | 2.000 |
| 5 | $b_{1}$ | -12.056 | 2.000 |
| 6 | $a_{1}$ | -10.884 | 2.000 |
| 7 | $b_{2}$ | -9.924 | 2.000 |
| 8 | $b_{1}$ | -6.190 | 2.000 |
| 9 | $b_{2}$ | -2.569 | 0.000 |
| 10 | $a_{1}$ | 1.918 | 0.000 |
| 11 | $b_{1}$ | 3.613 | 0.000 |
| 13 | $b_{2}$ | 9.266 | 0.000 |


$8 b_{1}$
$7 b_{2}$

$9 b_{2}$


However, the $\operatorname{HOMO}\left(b_{1}\right)$ to $\operatorname{NLUMO}\left(a_{1}\right)$ is allowed. In that case the excitation radiation must be polarized along $\times\left(b_{1}\right)$, since

$$
b_{1} b_{1} a_{1}=a_{1}
$$

Other allowed transitions include 7 -> 9 and 8 -> 11. The direct products are,

$$
b_{1} b_{1}=a_{1} \text { are } b_{2} b_{2}=a_{1}
$$

Note that a1 corresponds to z-polarization in $\mathrm{C}_{2 v}$. Thus, these products tells us that the $\pi-\pi^{*}$ transition and $n-p$ transition are allowed.

| $\mathrm{C}_{2 \mathrm{v}}$ | E | $\mathrm{C}_{2}$ | $\sigma_{\mathrm{v}(\mathrm{zz})}$ | $\sigma_{\mathrm{v}(\mathrm{yz})}$ | Rot | IR | Raman |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | 1 | 1 | 1 | 1 |  | z | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$ |
| $\mathrm{~A}_{2}$ | 1 | 1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ |  |  |
| $\mathrm{B}_{1}$ | 1 | -1 | 1 | -1 | $\mathrm{R}_{\mathrm{y}}$ | x |  |
| $\mathrm{B}_{2}$ | 1 | -1 | -1 | 1 | $\mathrm{R}_{\mathrm{x}}$ | y |  |

$$
\begin{array}{cccc}
1 & a & -509.064 & 2.000 \\
2 & b & -380.888 & 2.000 \\
3 & a & -380.888 & 2.000 \\
4 & a & -272.038 & 2.000 \\
5 & a & -25.833 & 2.000 \\
6 & a & -22.814 & 2.000 \\
7 & b & -21.872 & 2.000 \\
8 & a & -15.056 & 2.000 \\
9 & b & -13.468 & 2.000 \\
10 & a & -13.139 & 2.000 \\
11 & b & -11.366 & 2.000 \\
12 & b & -10.178 & 2.000 \\
13 & a & -9.805 & 2.000 \\
14 & a & -6.604 & 2.000 \\
15 & b & -6.530 & 2.000 \\
16 & b & -5.789 & 2.000 \\
17 & b & 0.135 & 0.000 \\
18 & a & 0.652 & 0.000
\end{array}
$$



## Cis and trans butadiene

cis

trans

$C_{2 h}$

## Trans butadiene Molecular orbitals

| $\mathrm{C}_{2 \mathrm{~h}}$ | E | $\mathrm{C}_{2}$ | i | $\sigma_{\mathrm{h}}$ | linear | quadratic |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{g}}$ | 1 | 1 | 1 | 1 | $\mathrm{R}_{\mathrm{z}}$ | $\mathrm{x}^{2}, \mathrm{y}^{2}, \mathrm{z}^{2}$, <br> xy |
| $\mathrm{B}_{\mathrm{g}}$ | 1 | -1 | 1 | -1 | $\mathrm{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}$ | $\mathrm{xz}, \mathrm{yz}$ |
| $\mathrm{A}_{\mathrm{u}}$ | 1 | 1 | -1 | -1 | z |  |
| $\mathrm{B}_{\mathrm{u}}$ | 1 | -1 | -1 | 1 | $\mathrm{x}, \mathrm{y}$ |  |

$b_{g}$

$a_{u}$ | $\\| \uparrow$ |
| :--- |
| $\downarrow$ |

$a_{u}$
$b_{g}$


| MO |  |  | Irrep |
| :---: | :--- | :--- | :--- |
| 1 | $\mathrm{a}_{\mathrm{g}}$ | -269.085 | Occupation |
| 2 | $\mathrm{~b}_{\mathrm{u}}$ | -269.078 | 2.000 |
| 3 | $\mathrm{a}_{\mathrm{g}}$ | -268.876 | 2.000 |
| 4 | $\mathrm{~b}_{\mathrm{u}}$ | -268.875 | 2.000 |
| 5 | $\mathrm{a}_{\mathrm{g}}$ | -19.916 | 2.000 |
| 6 | $\mathrm{~b}_{\mathrm{u}}$ | -18.033 | 2.000 |
| 7 | $\mathrm{a}_{\mathrm{g}}$ | -14.871 | 2.000 |
| 8 | $\mathrm{~b}_{\mathrm{u}}$ | -13.504 | 2.000 |
| 9 | $\mathrm{~b}_{\mathrm{u}}$ | -11.354 | 2.000 |
| 10 | $\mathrm{a}_{\mathrm{g}}$ | -11.276 | 2.000 |
| 11 | $\mathrm{a}_{\mathrm{g}}$ | -9.463 | 2.000 |
| 12 | $\mathrm{~b}_{\mathrm{u}}$ | -9.299 | 2.000 |
| 13 | $\mathrm{a}_{\mathrm{g}}$ | -8.308 | 2.000 |
| 14 | $\mathrm{a}_{\mathrm{u}}$ | -7.962 | 2.000 |
| 15 | $\mathrm{~b}_{\mathrm{g}}$ | -5.719 | 2.000 |
| 16 | $\mathrm{a}_{\mathrm{u}}$ | -1.783 | 0.000 |
| 17 | $\mathrm{~b}_{\mathrm{g}}$ | 0.850 | 0.000 |
| 18 | $\mathrm{~b}_{\mathrm{u}}$ | 1.932 | 0.000 |



## Determining the allowed transitions of trans butadiene

| $\mathrm{C}_{2 \mathrm{~h}}$ | E | $\mathrm{C}_{2}$ | i | $\sigma_{\mathrm{h}}$ | linear |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{\mathrm{u}}$ | 1 | 1 | -1 | -1 | z |
| $\mathrm{B}_{\mathrm{g}}$ | 1 | -1 | 1 | -1 | $\mathrm{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}$ |
| $\mathrm{B}_{\mathrm{u}}$ | 1 | -1 | -1 | 1 | $\mathrm{x}, \mathrm{y}$ |

The direct product of $A_{u}$ and $B_{g}$ is $B_{u}$. This means that transitions are allowed along both the x and y polarizations.

## Benzene

Most of the spectroscopy and reactivity of benzene is attributable to its $\pi$ system. To a good approximation the $\sigma$ and $\pi$ orbitals can be separated.

We consider how the 6 p-orbitals transform under $\mathrm{D}_{6 \mathrm{~h}}$ symmetry.

In this case we do not need to construct SALCs since there is only one type of orbital.


| $\mathrm{D}_{6 \mathrm{~h}}$ | E | $2 \mathrm{C}_{6}$ | $2 \mathrm{C}_{3}$ | $\mathrm{C}_{2}$ | $3 \mathrm{C}_{2}^{\prime}$ | $3 \mathrm{C}_{2}^{\prime \prime}$ | i | $2 \mathrm{~S}_{3}$ | $2 \mathrm{~S}_{6}$ | $*_{\mathrm{h}}$ | $3 *_{\mathrm{v}}$ | $3 *_{\mathrm{d}}$ | Irreps |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Gamma$ | 6 | 0 | 0 | 0 | -2 | 0 | 0 | 0 | 0 | -6 | 2 | 0 | $\mathrm{a}_{2 \mathrm{u}}+\mathrm{b}_{2 \mathrm{~g}}+$ |
| $\mathrm{e}_{1 \mathrm{~g}}+\mathrm{e}_{2 \mathrm{u}}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |

## $D_{6 h}$ character table

|  | E | $2 \mathrm{C}_{6}$ | $2 \mathrm{C}_{3}$ | $\mathrm{C}_{2}$ | $3 \mathrm{C}_{2}$ | $3 \mathrm{C}_{2}^{\prime \prime}$ | i | $2 \mathrm{~S}_{3}$ | $2 \mathrm{~S}_{6}$ | $\sigma_{\mathrm{h}}$ | $3 \sigma_{\mathrm{d}}$ | $3 \sigma_{v}$ | Linear | Quad |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1 \mathrm{~g}}$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{x}^{2}+\mathrm{y}^{2}, \mathrm{z}^{2}$ |
| $\mathrm{~A}_{2 \mathrm{~g}}$ | 1 | 1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | $\mathrm{R}_{\mathrm{z}}$ |  |
| $\mathrm{B}_{1 \mathrm{~g}}$ | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |  |  |
| $\mathrm{~B}_{2 \mathrm{~g}}$ | 1 | -1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |  |  |
| $\mathrm{E}_{1 \mathrm{~g}}$ | 2 | 1 | -1 | -2 | 0 | 0 | 2 | 1 | -1 | -2 | 0 | 0 | $\left(\mathrm{R}_{\mathrm{x}}, \mathrm{R}_{\mathrm{y}}\right)$ | $(\mathrm{xz}, \mathrm{yz})$ |
| $\mathrm{E}_{2 \mathrm{~g}}$ | 2 | -1 | -1 | 2 | 0 | 0 | 2 | -1 | -1 | 2 | 0 | 0 |  | $\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right.$, |
| $\mathrm{A}_{\mathrm{lu}}$ | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 |  |  |
| $\mathrm{~A}_{2 \mathrm{u}}$ | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | z |  |
| $\mathrm{B}_{1 \mathrm{u}}$ | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 |  |  |
| $\mathrm{~B}_{2 \mathrm{u}}$ | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 |  |  |
| $\mathrm{E}_{1 \mathrm{u}}$ | 2 | 1 | -1 | -2 | 0 | 0 | -2 | -1 | 1 | 2 | 0 | 0 | $(x, y)$ |  |
| $\mathrm{E}_{2 u}$ | 2 | -1 | -1 | 2 | 0 | 0 | -2 | 1 | 1 | -2 | 0 | 0 |  |  |

## Projection operator approach

The operation required to carry the reference p1 orbital into any of the others.

| Atom | Operation | $\mathrm{a}_{2 \mathrm{u}}$ | $\mathrm{b}_{2 \mathrm{~g}}$ | $\mathrm{e}_{1 \mathrm{~g}}$ | $\mathrm{e}_{2 \mathrm{u}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | E | 1 | 1 | 2 | 2 |
| 2 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |
| 3 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 4 | $\mathrm{C}_{2}$ | 1 | -1 | -2 | 2 |
| 5 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 6 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |



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The operation required to carry the reference p1 orbital into any of the others.

| Atom | Operation | $\mathrm{a}_{2 \mathrm{u}}$ | $\mathrm{b}_{2 \mathrm{~g}}$ | $\mathrm{e}_{1 \mathrm{~g}}$ | $\mathrm{e}_{2 \mathrm{u}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | E | 1 | 1 | 2 | 2 |
| 2 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |
| 3 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 4 | $\mathrm{C}_{2}$ | 1 | -1 | -2 | 2 |
| 5 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 6 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |



## Projection operator approach

The operation required to carry the reference p1 orbital into any of the others.

| Atom | Operation | $\mathrm{a}_{2 \mathrm{u}}$ | $\mathrm{b}_{2 \mathrm{~g}}$ | $\mathrm{e}_{1 \mathrm{~g}}$ | $\mathrm{e}_{2 \mathrm{u}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | E | 1 | 1 | 2 | 2 |
| 2 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |
| 3 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 4 | $\mathrm{C}_{2}$ | 1 | -1 | -2 | 2 |
| 5 | $\mathrm{C}_{3}$ | 1 | 1 | -1 | -1 |
| 6 | $\mathrm{C}_{6}$ | 1 | -1 | 1 | -1 |



## Molecular orbitals constructed using the projection operator approach

$$
\begin{gathered}
\Psi_{\mathrm{b}_{2 \mathrm{~g}}}=\frac{1}{\sqrt{6}}\left(\mathrm{p}_{1}-\mathrm{p}_{2}+\mathrm{p}_{3}-\mathrm{p}_{4}+\mathrm{p}_{5}-\mathrm{p}_{6}\right) \\
\Psi_{\mathrm{e}_{2 \mathrm{u}}}=\frac{1}{\sqrt{12}}\left(2 \mathrm{p}_{1}-\mathrm{p}_{2}-\mathrm{p}_{3}+2 \mathrm{p}_{4}-\mathrm{p}_{5}-\mathrm{p}_{6}\right) \\
\Psi_{\mathrm{e}_{1 \mathrm{~g}}}=\frac{1}{\sqrt{12}}\left(2 \mathrm{p}_{1}+\mathrm{p}_{2}-\mathrm{p}_{3}-2 \mathrm{p}_{4}-\mathrm{p}_{5}+\mathrm{p}_{6}\right) \\
\Psi_{\mathrm{a}_{2 \mathrm{u}}}=\frac{1}{\sqrt{6}}\left(\mathrm{p}_{1}+\mathrm{p}_{2}+\mathrm{p}_{3}+\mathrm{p}_{4}+\mathrm{p}_{5}+\mathrm{p}_{6}\right)
\end{gathered}
$$





## Benzene transitions

In benzene both the HOMO and LUMO are doubly degenerate. This is a common pattern observed in all aromatic molecules. The allowed transitions will have

$$
\Gamma_{\text {homo }} \Gamma_{\text {x,y or } z} \Gamma_{\text {lumo }}=\mathrm{A}_{1 \mathrm{~g}}
$$

Since it can be tedious to perform separate calculations for $x, y$ and $z$, we can also use the shortcut:

$$
\Gamma_{\text {homo }} \Gamma_{\text {lumo }}=\Gamma_{x, y \text { or } z}
$$

$M O$
7
eV
-21.136
-18.361
-14.759
-12.839 2.000
-11.145 2.000
-10.844 2.000
-10.189 2.000
-8.946 2.000
-8.204 2.000
$-6.231 \quad 2.000$
-1.065 0.000
$1.500 \quad 0.000$
2.8570 .000
2.9270 .000
$4.039 \quad 0.000$

## Direct product of HOMO-LUMO transition

$$
\begin{array}{ccccccccccccc} 
& \mathrm{E} & 2 \mathrm{C}_{6} & 2 \mathrm{C}_{3} & \mathrm{C}_{2} & 3 \mathrm{C}_{2}^{\prime} & 3 \mathrm{C}_{2}^{\prime \prime} & \mathrm{i} & 2 \mathrm{~S}_{3} & 2 \mathrm{~S}_{6} & \sigma_{\mathrm{h}} & 3 \sigma_{\mathrm{d}} & 3 \sigma_{\mathrm{v}} \\
& & & & & & & & & & 0 & 0 & 2 \\
1 & 1 & -1 & -2 & 0 & 0 \\
\mathrm{E}_{1 \mathrm{~g}} & 2 & 1 & -1 & -2 & 0 & & & & & & & \\
\mathrm{E}_{2 \mathrm{u}} & 2 & -1 & -1 & 2 & 0 & 0 & -2 & 1 & 1 & -2 & 0 & 0 \\
\Gamma & 4 & -1 & 1 & -4 & 0 & 0 & -4 & 1 & -1 & 4 & 0 & 0 \\
\hline
\end{array}
$$

The transition will be allowed if the direct product of the HOMO and LUMO contains one of the irreps corresponding to $\mathrm{x}, \mathrm{y}$ or z .

$$
\Gamma=b_{1 u}+b_{2 u}+e_{1 u}
$$

Since $e_{1 u}$ transforms as $x, y$, this transition is an allowed transition. Benzene is a planar absorber meaning that the absorbance is equal along the $x$ and $y$ directions. However, the fact that the irreps also include $b_{1 u}$ and $b_{2 u}$ means that this transition can also be vibronically allowed.

## Vibronically allowed transitions

For benzene we see that there are allowed transitions from the HOMO ( $e_{1 g}$ ) to LUMO ( $e_{2 u}$ ). However, if the symmetry of the molecule is lowered by vibration along an appropriate symmetry normal mode then transitions are also possible:

$$
\Gamma_{\text {homo }} \Gamma_{\text {vib }} \Gamma_{\mathrm{x}, \mathrm{y} \text { or } z} \Gamma_{\text {lumo }}=\mathrm{A}_{\text {tg }}
$$

For example, in the $e_{1 g}->e_{2 u}$ transition of benzene there can also be a transition that would normally be forbidden. When there are many direct products to compute it can be handy to use a product table. The next slide shows the $\mathrm{D}_{6 \mathrm{~h}}$ product table.

## Product table for the $\mathrm{D}_{6 \mathrm{~h}}$ point group

|  | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{A}_{\text {lu }}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{24}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 u}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 u}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ |
| $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{14}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 u}$ | $\mathrm{E}_{14}$ | $\mathrm{E}_{2 u}$ |
| $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2} \mathrm{~g}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ |
| $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{14}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ |
| $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{E}_{2 \mathrm{~g}}$ | $\begin{gathered} \mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}} \\ +\mathrm{E}_{1 \mathrm{~g}} \end{gathered}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\underset{\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+}{\mathrm{E}_{2 \mathrm{l}}}$ | $\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}+\mathrm{E}_{1 \mathrm{u}}$ |
| $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}+\mathrm{E}_{1 \mathrm{~g}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}} \\ +\mathrm{E}_{2 \mathrm{~g}} \end{gathered}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\begin{gathered} \mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}+ \\ \mathrm{E}_{1 \mathrm{u}} \\ \hline \end{gathered}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{E}_{2 \mathrm{u}}$ |
| $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{A}_{\text {lu }}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{B}_{14}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{1}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 g}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2}$ g |
| $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{l}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ |
| $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ |
| $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{E}_{2}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ |
| $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{E}_{2 \mathrm{u}}$ | $\begin{gathered} \mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}} \\ +\mathrm{E}_{1 \mathrm{u}} \end{gathered}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mid$ | $\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}+\mathrm{E}_{1 \mathrm{~g}}$ |
| $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{2 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{E}_{1 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}+\mathrm{E}_{1 \mathrm{u}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}} \\ +\mathrm{E}_{2 \mathrm{u}} \end{gathered}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{2 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\mathrm{E}_{1 \mathrm{~g}}$ | $\underset{\mathrm{E}_{1 \mathrm{~g}}}{\mathrm{~B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}+}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{E}_{2 \mathrm{~g}}$ |

## Pyridine

Pyridine has many of the spectroscopic characteristics of Benzene. However, pyridine has the lone pair of the N atom, Which leads to the possibility of $n-\pi^{*}$ transitions in addition To the $\pi-\pi^{\star}$ transitions of benzene. Pyridine belongs to the $C_{2 v}$ point group.


| $\mathrm{C}_{2 \mathrm{v}}$ | E | $\mathrm{C}_{2}$ | $\sigma_{\mathrm{v}(\mathrm{xz})}$ | $\sigma_{\mathrm{v}(\mathrm{yz})}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | 1 | 1 | 1 | 1 |
| $\mathrm{~A}_{2}$ | 1 | 1 | -1 | -1 |
| $\mathrm{~B}_{1}$ | 1 | -1 | 1 | -1 |
| $\mathrm{~B}_{2}$ | 1 | -1 | -1 | 1 |



Pyridine transitions MO Irrep eV Occupation

The $n-\pi^{*}$ is:
$21 a_{1}->22 b_{2}$
Weak absorption 280 nm
The $\pi-\pi^{\star}$ transitions are similar to benzene:
$19 b_{2}->22 b_{2}$ or $23 a_{2}$ $20 a_{2}->22 b_{2}$ or $23 a_{2}$

Moderate absorption
274 nm, 230 nm
Strong absorption 169 nm

| 11 | $a_{1}$ | -15.520 | 2.000 |
| :---: | :---: | :---: | :---: |
| 12 | $a_{1}$ | -13.042 | 2.000 |
| 13 | $b_{1}$ | -11.820 | 2.000 |
| 14 | $a_{1}$ | -11.487 | 2.000 |
| 15 | $b_{1}$ | -10.459 | 2.000 |
| 16 | $b_{2}$ | -9.824 | 2.000 |
| 17 | $a_{1}$ | -9.816 | 2.000 |
| 18 | $b_{1}$ | -8.800 | 2.000 |
| 19 | $b_{2}$ | -7.222 | 2.000 |
| 20 | $a_{2}$ | -6.623 | 2.000 |
| 21 | $a_{1}$ | -5.896 | 2.000 |
| 22 | $b_{2}$ | -1.778 | 0.000 |
| 23 | $a_{2}$ | -1.412 | 0.000 |
| 24 | $a_{1}$ | 1.519 | 0.000 |
| 25 | $b_{2}$ | 2.421 | 0.000 |

## Product table for the $\mathrm{C}_{2 \mathrm{v}}$ point group

We can see that there are numerous possibilities for transitions. Using the method that

$$
\Gamma_{\text {homo }} \Gamma_{\text {lumo }}=\Gamma_{x, y \text { or } z}
$$

for an allowed transition, and recalling that $\mathrm{x}, \mathrm{y}$ and z transform as $\mathrm{b}_{1}, \mathrm{~b}_{2}$ and $\mathrm{a}_{1}$, respectively, we can quickly determine the allowed transitions and polarizations using a $C_{2 v}$ product table.

|  | $\mathrm{A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~B}_{1}$ | $\mathrm{~B}_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{1}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~B}_{1}$ | $\mathrm{~B}_{2}$ |
| $\mathrm{~A}_{2}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{1}$ | $\mathrm{~B}_{2}$ | $\mathrm{~B}_{1}$ |
| $\mathrm{~B}_{1}$ | $\mathrm{~B}_{1}$ | $\mathrm{~B}_{2}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A}_{2}$ |
| $\mathrm{~B}_{2}$ | $\mathrm{~B}_{2}$ | $\mathrm{~B}_{1}$ | $\mathrm{~A}_{2}$ | $\mathrm{~A}_{1}$ |

