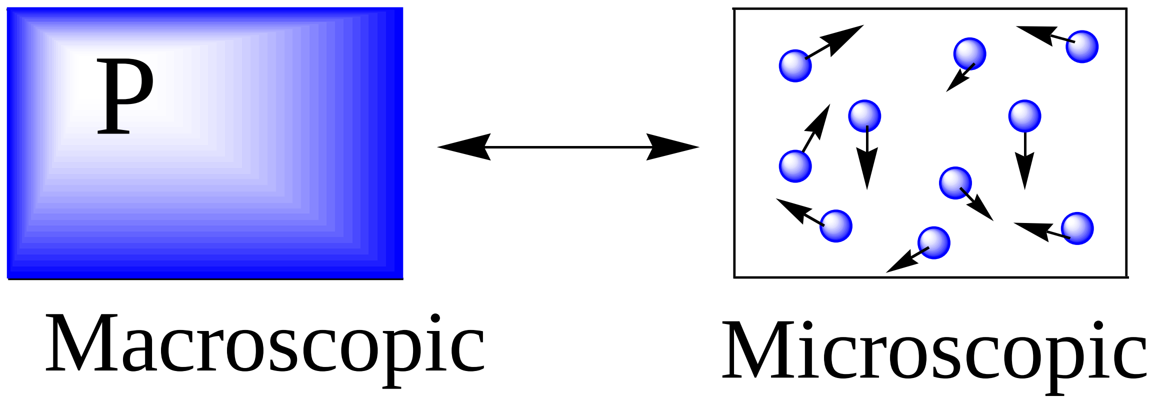
Equation of state relates P, V and T

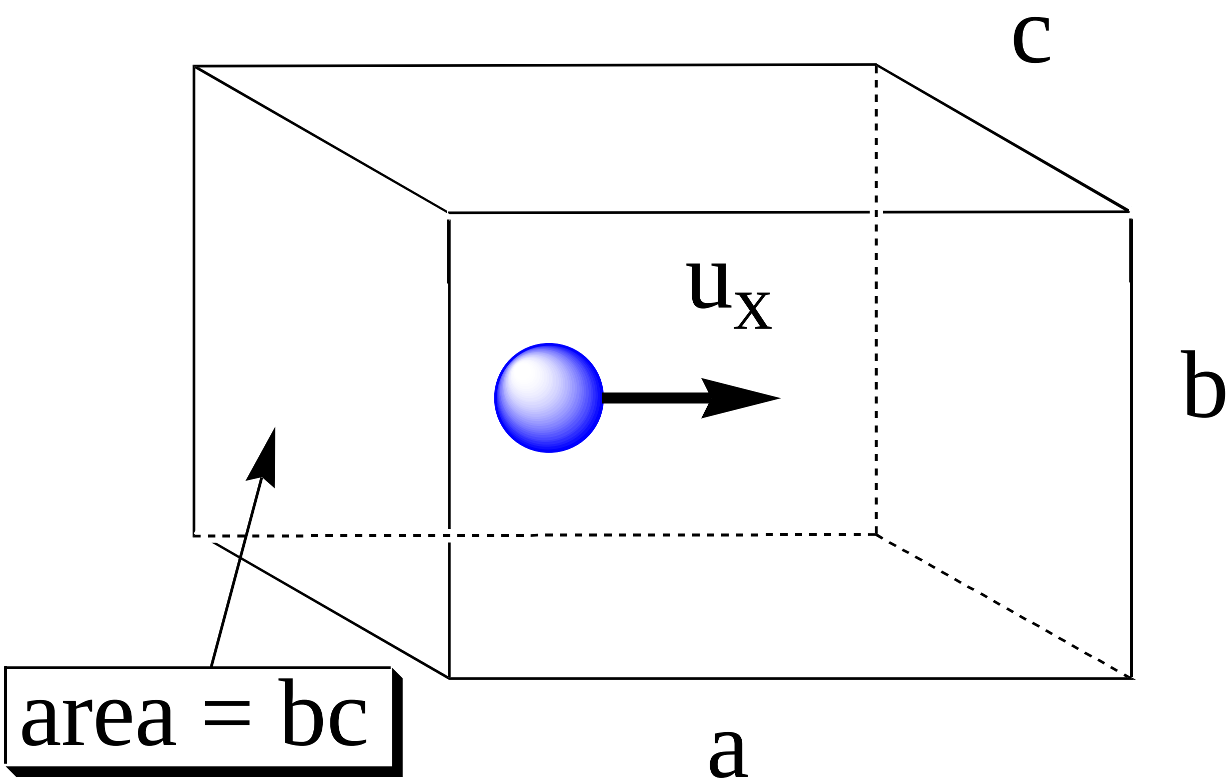
The ideal gas equation of state is

$$PV = nRT$$

An equation of state relates macroscopic properties which result from the average behavior of a large number of particles.



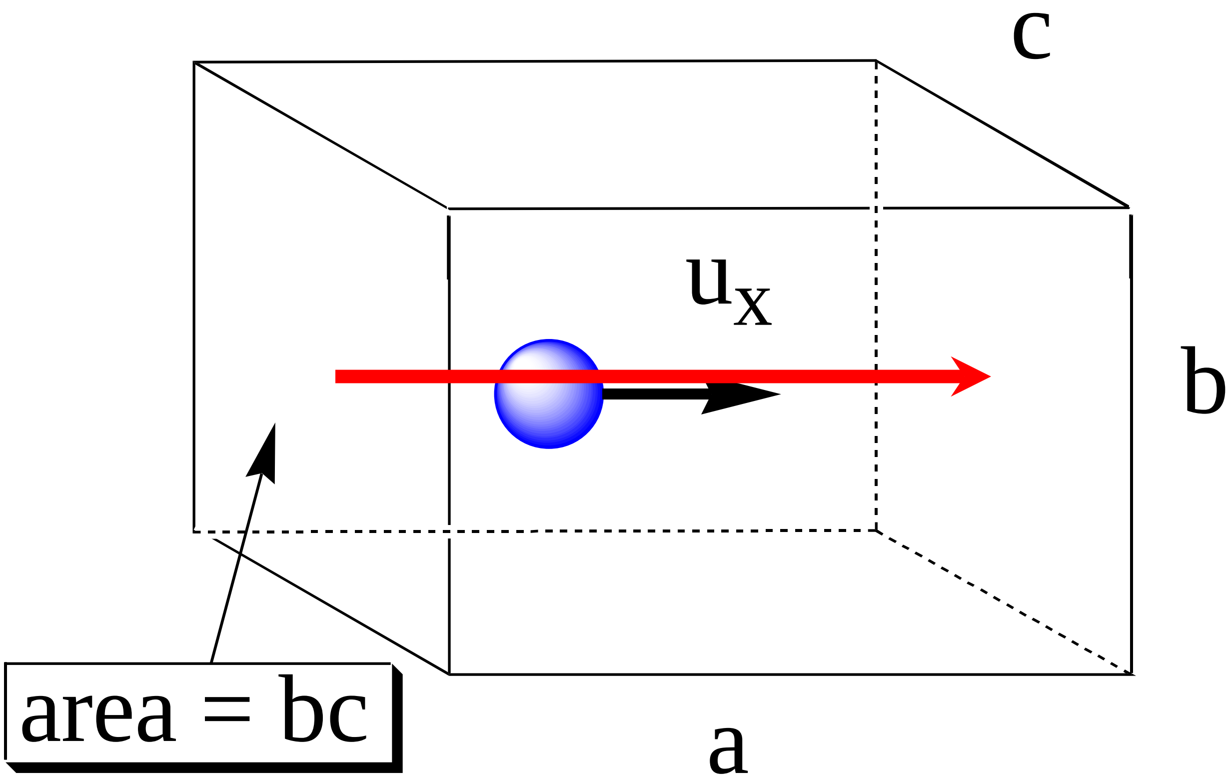
Microsopic view of momentum



A particle with velocity u_x strikes a wall.

The momentum of the particle changes from mu_x to $-mu_x$. The momentum change is $\Delta p = 2mu_x$.

Transit time

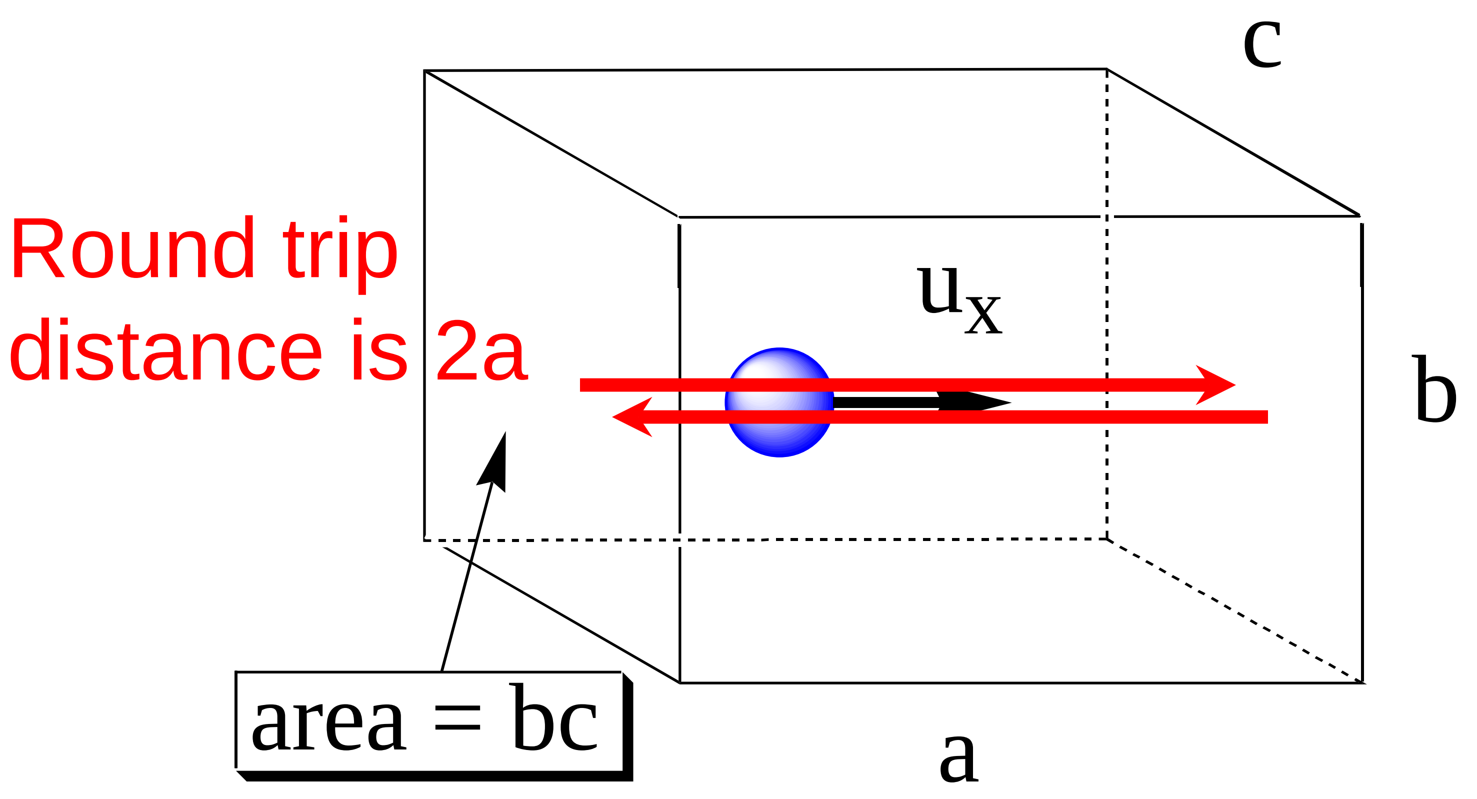


The time between collisions with one wall is

$\Delta t = 2a/u_x$.

This is also the round trip time.

Transit time



The time between collision is $\Delta t = 2a/u_x$.

velocity = distance/time.

time = distance/velocity.

The pressure on the wall

force = rate of change of momentum

$$F = \frac{\Delta p}{\Delta t} = \frac{2mu_x}{2a/u_x} = \frac{mu_x^2}{a}$$

The pressure is the force per unit area.

The area is A = bc and the

volume of the box is V = abc

$$P = \frac{F}{bc} = \frac{mu_x^2}{abc} = \frac{mu_x^2}{V}$$

Average properties

Pressure does not result from a single particle striking the wall but from many particles. Thus, the velocity is the average velocity times the number of particles.

$$P = \frac{Nm\langle u_x^2 \rangle}{V}$$

$$PV = Nm\langle u_x^2 \rangle$$

Average properties

There are three dimensions so the velocity along the x-direction is 1/3 the total.

$$\langle u_x^2 \rangle = \frac{1}{3}\langle u^2 \rangle$$

$$PV = \frac{Nm\langle u^2 \rangle}{3}$$

From the kinetic theory of gases

$$\frac{1}{2}Nm\langle u^2 \rangle = \frac{3}{2}nRT$$

Putting the results together

When we combine of microscopic view of pressure with the kinetic theory of gases result we find the ideal gas law.

$$PV = nRT$$

This approach assumes that the molecules have no size (take up no space) and that they have no interactions.