Solubility of gases

Henry's law constants in H_2O (atm x 10^3)	
He	131
N_2	86
CO	57
O_2	43
Ar	40
CO_2	1.6

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Solution:

$$x_{CO_2} = \frac{P_{CO_2}}{K_{H,CO_2}}$$

$$x_{CO_2} = \frac{3.8\ x\ 10^{-4}\ atm}{1600\ atm} = 2.375\ x\ 10^{-7}$$

$$c_{CO_2} \approx x_{CO_2} c_{H_2O} = (2.375\ x\ 10^{-7})(55.5\ M)$$

$$c_{CO_2} \approx 1.31\ x\ 10^{-5}$$

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The carbonic acid equilibrium is

$$CO_2 + H_2O \rightarrow HCO_3^- + H^+$$

The K_a is

$$K_a = 10^{-pK_a} = 10^{-6.3} = 5.01\,x\,10^{-6}$$

Given that the initial concentration is quite low,

$$c_{CO_2} \approx 1.31\,x\,10^{-5}$$

it would be best to solve for the $[H^+]$ using a reaction table.

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Accounting for the $[H^+] = 10^{-7}$ in neutral water we have

$$K_a = \frac{x(10^{-7} + x)}{(c_{CO_2} - x)}$$

$$K_a\left(c_{CO_2} - x\right) = 10^{-7}x + x^2$$

$$x^2 + (K_a + 10^{-7})x - K_a c_{CO_2} = 0$$

$$x^2 + (5.11\ x\ 10^{-6})x - 6.56\ x\ 10^{-11} = 0$$

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which leads to

$$x = \frac{-5.11\, x\, 10^{-6} \pm \sqrt{2.61\, x\, 10^{-11} + 4(6.56\, x\, 10^{-11})}}{2}$$

$$x = 5.93\, x\, 10^{-6}$$

and therefore pH = 5.2.