# Formation of black phosphorous

There are three allotropes of phosphorous, white, violet and black, which have densities of 1.88, 2.36 and 2.69 gm/cm$^3$, respectively. At ambient pressure white phosphorous is stable. Given that the free energy for formation of black phosphorous from white phosphorous is 2.7 kJ/mol, what pressure is required to convert white phosphorous to black phosphorous?

$$P_4 \text{ (white)} \longrightarrow P_n \text{ (black)}$$

NOTE: For any density calculations treat the phosphorous in both species as atomic phosphorous with a mass of 31 a.m.u. Furthermore, for this problem assume that Q = 1, which means that the pressure you calculate will be sufficient to begin to convert white phosphorous to black phosphorous.

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Solution:

$$\Delta G = \Delta G^o + RT\ lnQ$$

However, since Q = 0, $\Delta G = \Delta G^o$. In order to force the reaction to occur against a positive $\Delta G^o$ we use pressure according to

$$\Delta G = \Delta V_m \Delta P$$

Therefore, $\Delta G$ = - 2.7 kJ/mol.

$$\Delta V_m = \frac{M_m}{\rho_{aq}} - \frac{M_m}{\rho_s}$$

$$\Delta V_m = \frac{0.031\ kg/mol}{2690\ kg/m^3} - \frac{0.031\ kg/mol}{1880\ kg/m^3}$$

$$\Delta V_m = -5.1\ x\ 10^{-6}\ m^3/mol$$

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The pressure required to force the formation of black phosphorous to occur is

$$\mathrm{P} = 10^5 Pa + \frac{\Delta G}{\Delta V_m}$$

$$\mathrm{P} = 10^5 Pa \; + \frac{-2{,}700\ J/mol}{-5.1\ x\ 10^{-6}\ m^3/mol}$$

$$P = 5.27\ x\ 10^8\ Pa = 5270\ bars$$