Equilibrium of smog formation

Pressure dependence

$$N_2O_4(g) \rightarrow 2\ NO_2(g)$$

We can write the equilibrium constant:

$$K = \frac{{P_{NO_2}}^2}{P_{N_2O_4}}$$

If there is a constraint on the total pressure then We must use Dalton's law to calculate the mole Fractions.

$$P_{N_2O_4} = x_{N_2O_4} P_{tot}$$

$$P_{NO_2} = x_{NO_2} P_{tot}$$

Pressure dependence

$$N_2O_4(g) \rightarrow 2\,NO_2(g)$$

The equilibrium constant is

$$K = \frac{{x_{NO_2}}^2}{x_{N_2O_4}} P_{tot}$$

For a given initial pressure of N_2O_4 we have

	N_2O_4	NO_2	Total
Initial	$P^0_{N_2O_4}$	0	$P^0_{N_2O_4}$
Delta	-x	2x	+x
Final	$P^0_{N_2O_4} - x$	$2x$	$P^0_{N_2O_4} + x$

To solve a gas phase problem you need to know whether the pressure can change or not. If it is fixed, then P_{tot} has a value that does not change. The mole fractions are,

$$x_{NO_2} = \frac{2x}{P^0_{N_2O_4} + x} \qquad x_{N_2O_4} = \frac{P^0_{N_2O_4} - x}{P^0_{N_2O_4} + x}$$

For example, if we assume that the initial pressure is maintained, then the above values should be substituted into the equilibrium constant

$$K = \frac{{x_{NO_2}}^2}{x_{N_2O_4}} P_{tot}$$

$$K = \frac{\left(\frac{2x}{P^0_{N_2O_4} + x}\right)^2}{\frac{P^0_{N_2O_4} - x}{P^0_{N_2O_4} + x}} P^0_{N_2O_4}$$

$$K = \frac{\left(\frac{2x}{P^0_{N_2O_4} + x}\right)^2}{\frac{P^0_{N_2O_4} - x}{P^0_{N_2O_4} + x}} P^0_{N_2O_4}$$

$$K = \frac{P^0_{N_2O_4}(2x)^2}{(P^0_{N_2O_4} - x)(P^0_{N_2O_4} + x)}$$

$$K = \frac{\left(\frac{2x}{P^0_{N_2O_4} + x}\right)^2}{\frac{P^0_{N_2O_4} - x}{P^0_{N_2O_4} + x}} P^0_{N_2O_4}$$

$$K = \frac{P^0_{N_2O_4}(2x)^2}{\left(P^0_{N_2O_4} - x\right)\left(P^0_{N_2O_4} + x\right)}$$

$$K\left(P^0_{N_2O_4}\right)^2 - \left(K + 4P^0_{N_2O_4}\right)x^2 = 0$$

$$x = \sqrt{\frac{K\left(P^0_{N_2O_4}\right)^2}{\left(K + 4P^0_{N_2O_4}\right)}}$$