# Adiabatic Processes

If a process occurs in an isolated system then no heat can be transferred between the system and surroundings. In this case the heat transferred, q, is zero, i.e. q = 0. Therefore,

$$\Delta U = w$$

We call such processes adiabatic. Actually, this special case is of great importance. For example, when a column of air rises in the atmosphere it expands and cools adiabatically. Expressed in differential format:

$$dU = \delta w$$

# Adiabatic Processes

Using the definition of the internal energy in terms of the heat capacity and the work in pressure-volume terms, we can write

$$nC_{v,m}dT = -PdV$$

Next, we use the ideal gas law

$$nC_{v,m}dT = -\left(\frac{nRT}{V}\right)dV$$

and rearrange to obtain a differential equation

$$nC_{v,m}\frac{dT}{T} = -nR\frac{dV}{V}$$

# Adiabatic Processes

We can integrate to find

$$nC_{v,m}\int_{T_1}^{T_2}\frac{dT}{T} = -nR\int_{V_1}^{V_2}\frac{dV}{V}$$

$$nC_{v,m}\,ln\left(\frac{T_2}{T_1}\right) = -nR\,ln\left(\frac{V_2}{V_1}\right)$$

We absorb the minus sign for convenience

$$nC_{v,m}\,ln\left(\frac{T_2}{T_1}\right) = nR\,ln\left(\frac{V_1}{V_2}\right)$$

# Expansion of a diatomic gas

Using the form on the previous page we can derive the relationship between the volume change and temperature.

$$\left(\frac{T_2}{T_1}\right)^{nC_{v,m}} = \left(\frac{V_1}{V_2}\right)^{nR}$$

And

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_{v,m}}$$

For a diatomic gas we find

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{2/5}$$

This expression is great practical value since you can predict the temperature of air as it rises.  This phenomenon leads to rain over mountains and cooling that affects ecosystems at high elevation.

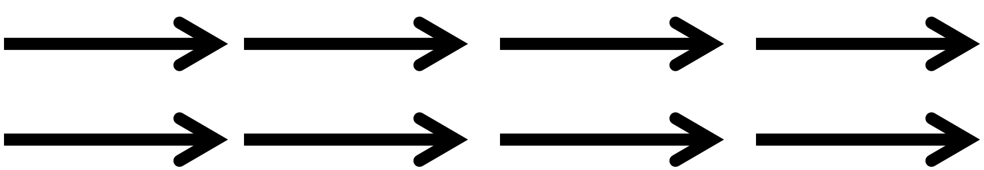
# Application to meterology

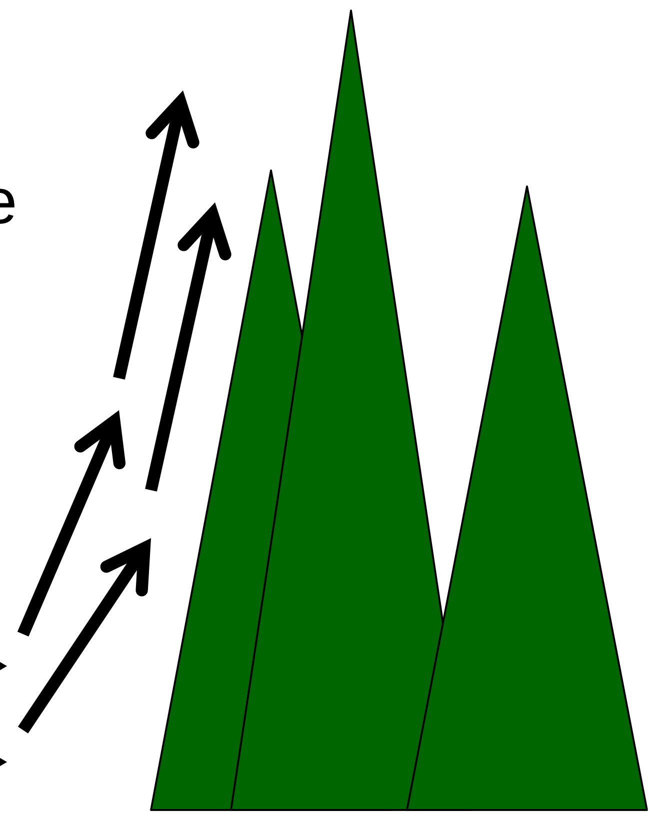
Elevation increases the average rainfall.  This occurs because air masses rise as the encounter mountains, and as they rise they cool.  Because the lateral heat transfer in the atmosphere is poor, we can treat this as an adiabatic cooling.

In this application we can calculate the pressure change

$$P = P_0 exp\left\{-\frac{Mgh}{RT}\right\}$$

but not the volume change with h.





# Pressure dependence of adiabatic expansion

$$PV = nRT$$

In an adiabatic expansion all of the variable, P, V and T can change.

$$\frac{P_2V_2}{P_1V_1} = \frac{nRT_2}{nRT_1}$$

Therefore,

$$\frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1} \longrightarrow \left(\frac{V_1}{V_2}\right)^{nR/C_v} = \frac{T_2}{T_1}$$

# Pressure dependence of adiabatic expansion

Solve for $V_1/V_2$ and substitute into the formula

$$\frac{P_2 T_1}{P_1 T_2} = \frac{V_1}{V_2} \quad \rightarrow \quad \left(\frac{V_1}{V_2}\right)^{nR/C_v} = \frac{T_2}{T_1}$$

$$\left(\frac{P_2 T_1}{P_1 T_2}\right)^{nR/C_v} = \frac{T_2}{T_1}$$

$$\left(\frac{P_2}{P_1}\right)^{nR/C_v} = \left(\frac{T_2}{T_1}\right)^{1+nR/C_v}$$

# Pressure dependence of adiabatic expansion

Rearrange the exponent to write it in a compact form:

$$\left(\frac{P_2}{P_1}\right)^{\frac{nR/C_v}{1+nR/C_v}} = \frac{T_2}{T_1}$$

$$\frac{nR/C_v}{1+nR/C_v} = \frac{nR}{C_v+nR} = \frac{nR}{C_p}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{nR}{C_p}}$$

# Example: Temperature on Mt. Mitchell on a summer day

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Solution:

Atmosphere is a diatomic gas so $C_p$ = 7/2nR.

Using the barometric pressure formula to find $P_2$ = 0.80 atm.

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{nR/Cp}$$

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$$T_2 = (310\ K)\left(\frac{0.8}{1}\right)^{\frac{2}{7}} = 290\ K$$

# The adiabatic pressure-volume formula

Since the variables, P, V and T all change in an adiabatic process, must use the following expression when substituting pressure for volume:

$$\frac{P_2 V_2}{P_1 V_1} = \frac{T_2}{T_1}$$

Thus, we can obtain the pressure volume relationship,

$$\frac{P_2 V_2}{P_1 V_1} = \left(\frac{V_1}{V_2}\right)^{R/C_{v,m}}$$

# The adiabatic pressure-volume formula

Rearranging we find

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{R/C_{v,m}} \left(\frac{V_1}{V_2}\right)$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{C_{v,m}}+1}$$

And this form is one of the final forms:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\frac{C_{p,m}}{C_{v,m}}}$$

# The adiabatic pressure-volume formula

It is customary to define

$$\gamma = \frac{C_{p,m}}{C_{v,m}}$$

so that the relation can be written

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

or alternatively as

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$