Adiabatic Processes

If a process occurs in an isolated system then no heat can be transferred between the system and surroundings. In this case the heat transferred, q, is zero, i.e. q = 0. Therefore,

$$\Delta U = w$$

We call such processes adiabatic. Actually, this special case is of great importance. For example, when a column of air rises in the atmosphere it expands and cools adiabatically. Expressed in differential format:

$$dU = \delta w$$

Adiabatic Processes

Using the definition of the internal energy in terms of the heat capacity and the work in pressure-volume terms, we can write

$$nC_{v,m}dT = -PdV$$

Next, we use the ideal gas law

$$nC_{v,m}dT = -\left(\frac{nRT}{V}\right)dV$$

and rearrange to obtain a differential equation

$$nC_{v,m}\frac{dT}{T} = -nR\frac{dV}{V}$$

Adiabatic Processes

We can integrate to find

$$nC_{v,m}\int_{T_1}^{T_2}\frac{dT}{T} = -nR\int_{V_1}^{V_2}\frac{dV}{V}$$

$$nC_{v,m}\,ln\left(\frac{T_2}{T_1}\right) = -nR\,ln\left(\frac{V_2}{V_1}\right)$$

We absorb the minus sign for convenience

$$nC_{v,m}\,ln\left(\frac{T_2}{T_1}\right) = nR\,ln\left(\frac{V_1}{V_2}\right)$$