## Approach to equilibrium

- $A \rightarrow B$ and $B \rightarrow A$

- Rate equations are:
- $\mathrm{d}[\mathrm{A}] / \mathrm{dt}=-\mathrm{k}_{1}[\mathrm{~A}]+\mathrm{k}_{1}[\mathrm{~B}]$
- $d[B] / d t=k_{1}[A]-k_{-1}[B]$
- Simultaneous solutions of these equations leads to a rate constant of $\mathrm{k}_{1}+\mathrm{k}_{-1}$.
- Equilibrium is approached as the sum of the forward and reverse rate constants.


## Approach to equilibrium

Let $x$ be the deviation from equilibrium
$x=[A]-[A]_{\text {eq }}=[B]_{\text {eq }}-[B]$

## Approach to equilibrium

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then $[A]=[A]_{\text {eq }}+x$ and $[B]=[B]_{e q}-x$
Rate equations are:
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then $[A]=[A]_{\text {eq }}+x$ and $[B]=[B]_{e q}-x$
Rate equations are:
$-d\left([A]_{e q}+x\right) / d t=k_{1}\left([A]_{e q}+x\right)-k_{-1}\left([B]_{e q}-x\right)$
and
$-\mathrm{d}[\mathrm{A}]_{\mathrm{eq}} / \mathrm{dt}=\mathrm{k}_{1}[\mathrm{~A}]_{\mathrm{eq}}-\mathrm{k}_{-1}[\mathrm{~B}]_{\mathrm{eq}}=0$
We can understand this because $[\mathrm{A}]_{\mathrm{eq}}$ is a constant. A second point is that

$$
K=\frac{k_{1}}{k_{-1}}=\frac{[B]_{e q}}{[A]_{e q}}
$$

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Therefore,
$-\mathrm{dx} / \mathrm{dt}=\left(\mathrm{k}_{1}+\mathrm{k}_{-1}\right) \mathrm{x}$

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$\mathrm{x}=[\mathrm{A}]-[\mathrm{A}]_{\mathrm{eq}}=[\mathrm{B}]_{\mathrm{eq}}-[\mathrm{B}]$
then $[A]=[A]_{\mathrm{eq}}+\mathrm{x}$ and $[\mathrm{B}]=[\mathrm{B}]_{\mathrm{eq}}-\mathrm{x}$
Rate equations are:
$-\mathrm{d}\left([\mathrm{A}]_{\mathrm{eq}}+\mathrm{x}\right) / \mathrm{dt}=\mathrm{k}_{1}\left([\mathrm{~A}]_{\mathrm{eq}}+\mathrm{x}\right)-\mathrm{k}_{-1}\left([\mathrm{~B}]_{\mathrm{eq}}-\mathrm{x}\right)$
and
$-\mathrm{d}[\mathrm{A}]_{\mathrm{eq}} / \mathrm{dt}=\mathrm{k}_{1}[\mathrm{~A}]_{\mathrm{eq}}-\mathrm{k}_{-1}[\mathrm{~B}]_{\mathrm{eq}}=0$
Therefore,
$-\mathrm{dx} / \mathrm{dt}=\left(\mathrm{k}_{1}+\mathrm{k}_{-1}\right) \mathrm{x}$
$x(\mathrm{t})=\mathrm{x}_{0} \exp \left\{-\left(\mathrm{k}_{1}+\mathrm{k}_{-1}\right) \mathrm{t}\right\}$
NOTE: This is not intuitive. Return to equilibirum occurs at the sum of the forward and reverse rate constants.

## Principle of microscopic reversibility

- $A \rightarrow B$ and $B \rightarrow A$

- The equilibrium constant for such a process can be related for the forward and reverse rate constants.

$$
K=\frac{k_{1}}{k_{-1}}
$$

- From before:

$$
k_{o b s}=k_{1}+k_{-1}
$$

