Examples: Weak acid and weak base

20.0 mL of 0.30 M NaH_2PO_4 was added to 20.0 mL of 0.30 M NaHS. What are the concentrations of all species at equilibrium? [The pK_a of $H_2PO_4^-$ and H_2S are 7.2 and 6.9]

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Solution: Look up the K_a for each reaction involved in this acid-base equilibrium.

$$HS^- + H_2PO_4^- \leftrightarrow HPO_4^{2-} + H_2S$$

$$H_2PO_4^- \leftrightarrow HPO_4^{2-} + H^+$$

$$HS^- + H^+ \leftrightarrow H_2S$$

$$K = \frac{K_a(H_2PO_4^-)}{K_a(H_2S)} = \frac{6.20\ x\ 10^{-8}}{1.26\ x\ 10^{-7}} = 0.49$$

$$K = \frac{[HPO_4^{2-}][H_2S]}{[HS^-][H_2PO_4^-]}$$

Examples: Weak acid and weak base

Species	HS^-	$H_2PO_4^-$	HPO_4^{2-}	H_2S
Initial	0.15	0.15	0.0	0.0
Difference	-x	-x	x	x
Final	0.15-x	0.15-x	x	x

$$K = \frac{x^2}{(0.15 - x)^2}$$

$$K(0.15 - x)^2 - x^2 = 0$$

$$0.0225K - 0.3Kx - (1 - K)x^2 = 0$$

$$0.011 - 0.147x - 0.51x^2 = 0$$

$$x = \frac{0.147 \pm \sqrt{0.147^2 + 4(0.011)(0.51)}}{-2(0.51)}$$

$$x = 0.0636$$

Examples: Weak acid and weak base

Species	HS^-	$H_2PO_4{}^-$	$HPO_4{}^{2-}$	H_2S
Initial	0.15	0.15	0.0	0.0
Difference	-x	-x	x	x
Final	0.15-x	0.15-x	x	x

Thus the concentrations are:

$$[HS^-] = [H_2PO_4^-] = 0.0864\ M$$

$$[H_2S] = [HPO_4^{2-}] = 0.0636\ M$$

Such mixtures form a double buffer. Both the $[HS^-]/[H_2S]$ and the $[HPO_4^{2-}]/[H_2PO_4^-]$ ratio can be used in the Hendersen-Hasselbach equation to predict the pH. Regardless of which buffer we choose we will obtain the same answer for the pH.

Examples: Weak acid and weak base

Species	HS^-	$H_2PO_4^-$	HPO_4^{2-}	H_2S
Initial	0.15	0.15	0.0	0.0
Difference	-x	-x	x	x
Final	0.15-x	0.15-x	x	x

Use the Hendersen-Hasselbach equation to predict the pH.

$$pH = 7.2 + log_{10}\left(\frac{0.064}{0.086}\right) = 7.06$$

We can check using other pK_a for H_2S.

$$pH = 6.9 + log_{10}\left(\frac{0.086}{0.064}\right) = 7.03$$

The small difference is due to rounding error.