## Chemistry 201

## Weak Acids and Bases

NC State University
know $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right]$ calc pH

## Strong Acid

## Strong <br> Base



## Weak Base

## know [OH ${ }^{1-}$ ] calc pOH

## pH of Weak Acids \& Bases

- How do we find $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right.$ ?
- How do acid and base strengths relate?


## Relationship of $\mathrm{pK}_{\mathrm{a}}$ and $\mathrm{pK} \mathrm{b}_{\mathrm{b}}$

$$
\begin{gathered}
H A+H_{2} O \leftrightarrow A^{-}+H_{3} O^{+} \quad K_{a}=\frac{\left[A^{-}\right]\left[H^{+}\right]}{[H A]} \\
A^{-}+H_{2} O \leftrightarrow H A+O H^{-} \quad K_{b}=\frac{[H A]\left[O H^{-}\right]}{\left[A^{-}\right]} \\
H_{2} O+H_{2} O \leftrightarrow O H^{-}+H_{3} O^{+} \quad K_{w}=\left[H^{+}\right]\left[O H^{-}\right] \\
\left(\frac{\left[A^{-}\right]\left[H^{+}\right]}{[H A]}\right)\left(\frac{[H A]\left[O H^{-}\right]}{\left[A^{-}\right]}\right)=\left[H^{+}\right]\left[O H^{-}\right] \\
K_{a} K_{b}=K_{w} \\
p K_{a}+p K_{b}=p K_{w}
\end{gathered}
$$

## Calculation of the pH of weak acids

For a generic weak acid dissociation in water to produce ions we can consider the general case,

$$
H A \leftrightarrow A^{-}+H^{+}
$$

in which the initial concentration of $[\mathrm{HA}]=\mathrm{C}$ and $\left[A^{-}\right]=\left[H^{+}\right]=0$. Then we make the following reaction table:

| Molecule | $H A$ | $A^{-}$ | $H^{+}$ |
| :--- | :---: | :---: | :---: |
| Initial | C | 0 | 0 |
| Change | -x | x | x |
| Equilibrium | $\mathrm{C}-\mathrm{x}$ | x | x |

## Using the $\mathrm{K}_{\mathrm{a}}$ to obtain $\mathrm{x}=\left[\mathrm{H}^{+}\right]$

We can substitute these values into $K_{a}$,

$$
K_{a}=\frac{x^{2}}{C-x}
$$

This can be formulated as a general quadratic equation.

$$
K_{a} C-K_{a} x-x^{2}=0
$$

Note that we do not necessarily need to use the quadratic Formula. If $\mathrm{C} \gg \mathrm{x}$ then we can use an approximation

$$
x \approx \sqrt{C K_{a}}
$$

Which is justified on the next slide.

## Justification of approximate solution

The general case has a solution

$$
x=\frac{K_{a} \pm \sqrt{K_{a}^{2}+4 C K_{a}}}{-2}
$$

If the concentration $C$ of the acid is sufficiently large, then we can neglect x relative to x , i.e. $\mathrm{C} \gg \mathrm{x}$. In this case,

$$
K_{a} \approx \frac{x^{2}}{C}
$$

and

$$
x \approx \sqrt{C K_{a}}
$$

## pH of Weak Acids

What is the pH of a 0.20 M solution

$$
\text { of } \mathrm{HC}_{2} \mathrm{H}_{3} \mathrm{O}_{2} \text { ? } \quad\left(\mathrm{pK}_{\mathrm{a}}=4.74\right)
$$

## pH of Weak Acids

## What is the pH of a 0.20 M solution of $\mathrm{HC}_{2} \mathrm{H}_{3} \mathrm{O}_{2}$ ? $\left(\mathrm{pK}_{\mathrm{a}}=4.74\right)$

Solution: Set $A=\mathrm{HC}_{2} \mathrm{H}_{3} \mathrm{O}_{2}$

$$
\mathrm{HA} \rightarrow \mathrm{H}^{+}+\mathrm{A}^{-}
$$

Step 1: Solve for $K_{a}=1.82 \times 10^{-5}$
Step 2: Make a reaction table

| Molecule | HA | $\mathrm{A}^{-}$ | $\mathrm{H}^{+}$ |
| :--- | :--- | :--- | :--- |
| Initial | 0.2 | 0 | 0 |
| Difference | -x | x | x |
| Equilibrium | $0.2-\mathrm{x}$ | x | x |

## pH of Weak Acids

## What is the pH of a 0.20 M solution of $\mathrm{HC}_{2} \mathrm{H}_{3} \mathrm{O}_{2}$ ? $\left(\mathrm{pK}_{\mathrm{a}}=4.74\right)$

Step 3: Solve for x

$$
\begin{gathered}
K_{a}=\frac{\left[A^{-}\right]\left[H^{+}\right]}{[H A]}=\frac{x^{2}}{0.2-x} \\
K_{a}(0.2-x)-x^{2}=0 \quad K_{a} 0.2-K_{a} x-x^{2}=0 \\
x=\frac{K_{a} \pm \sqrt{K_{a}{ }^{2}+4(0.2) K_{a}}}{-2} \\
x=\frac{1.82 \times 10^{-5} \pm \sqrt{\left(1.82 \times 10^{-5}\right)^{2}+0.8\left(1.82 \times 10^{-5}\right)}}{-2} \\
x=0.00189
\end{gathered}
$$

## pH of Weak Acids

## What is the pH of a 0.20 M solution of $\mathrm{HC}_{2} \mathrm{H}_{3} \mathrm{O}_{2}$ ? $\left(\mathrm{pK}_{\mathrm{a}}=4.74\right)$

Step 3. Calculate pH

$$
p H=-\log _{10}(0.00189)=2.72
$$

We could have used the approximate method for this acid since $x=0.00189$ and $C=0.2$. According to the approximate method.

$$
x \approx \sqrt{C K_{a}}=\sqrt{(0.2)\left(1.82 \times 10^{-5}\right)}=0.00190
$$

The difference from the exact method is less than 1\%.

## know [H calc pH

## Strong Acid

## know [OH ${ }^{1-}$ ] calc pOH

Strong
Base

## rxn table use $\mathrm{K}_{\mathrm{a}}$ get $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right]$ calc pH

Weak Acid

## Weak <br> Base

## pH of Weak Bases

## What is the pH of a 0.10 M solution of NaF ? ( $\mathrm{pK}_{\mathrm{a}}=3.14$ for HF )

Solution: Write down the conjugate base reaction

$$
\mathrm{F}^{-}+\mathrm{H}_{2} \mathrm{O} \leftrightarrow \mathrm{HF}+\mathrm{OH}^{-}
$$

Step 1: Solve for $\mathrm{K}_{\mathrm{b}}$

$$
K_{b}=10^{-p K b}=10^{-10.86}=1.38 \times 10^{-11}
$$

$$
\begin{aligned}
& p K b=p K w-p K a \\
& p K b=14-3.14=10.86
\end{aligned}
$$

## pH of Weak Bases

## NaF problem(contd) ( $\mathrm{pK} \mathrm{K}_{\mathrm{a}}=3.14$ for HF)

Step 2: Make a reaction table

| Molecule | $\mathrm{F}^{-}$ | HF | $\mathrm{OH}^{-}$ |
| :--- | :--- | :--- | :--- |
| Initial | 0.1 | 0 | 0 |
| Difference | -x | x | x |
| Equilibrium | $0.1-\mathrm{x}$ | x | x |

Step 3: Solve for $x$

$$
\begin{gathered}
K_{b} 0.1-K_{b} x-x^{2}=0 \\
x=\frac{K_{b} \pm \sqrt{K_{b}^{2}+4(0.1) K_{b}} \quad x=1.17 \times 10^{-6}}{-2} \\
x=\frac{1.38 \times 10^{-11} \pm \sqrt{\left(1.38 \times 10^{-11}\right)^{2}+0.4\left(1.38 \times 10^{-11}\right)}}{-2}
\end{gathered}
$$

## pH of Weak Bases

## NaF problem(contd) $\left(\mathrm{pK}_{\mathrm{a}}=3.14\right.$ for HF$)$

Step 4: Solve for pOH

$$
\mathrm{pOH}=-\log _{10}\left(1.17 \times 10^{-6}\right)=5.93
$$

Step 5: Convert to pH

$$
\begin{gathered}
p H+p O H=p K_{w} \\
p H=p K_{w}-p O H \\
p H=14-5.93=8.07
\end{gathered}
$$

## pH of Weak Bases

## NaF problem(contd) $\left(\mathrm{pK}_{\mathrm{a}}=3.14\right.$ for HF$)$

 Here again we could have used the approximate method and saved a lot of work.$$
x \approx \sqrt{C K_{b}}=\sqrt{(0.1)\left(1.38 \times 10^{-11}\right)}=1.17 \times 10^{-6}
$$

Since Kb is so small in this case the difference is less than one part in 1000.

## know [H calc pH

## Strong Acid

## know [OH ${ }^{1-}$ ] calc pOH

Strong
Base

## rxn table use $\mathrm{K}_{\mathrm{a}}$ get $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right]$ calc pH

Weak Acid

## Weak <br> Base

know $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right]$ calc pH

## Strong Acid

## rxn table use $\mathrm{K}_{\mathrm{a}}$ get $\left[\mathrm{H}_{3} \mathrm{O}^{1+}\right]$ calc pH

Strong
Base

## know [OH ${ }^{1-}$ ] calc pOH

## Weak <br> Base



