Chemistry 201

Density

Concentration

Molarity vs. Molality

Mole fraction

NC State University

Density

The density of a material is defined as the mass per unit volume. Density is often given in grams per cubic centimeter (cc), which is the same thing as grams per milliliter (mL).

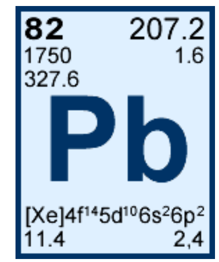
Density is a material property, ρ.

$\rho(H_2O)$ = 1 gm/mL.

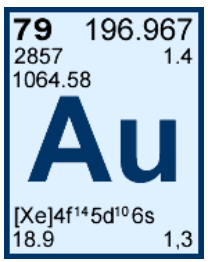
$\rho(C_2H_5OH)$ = 0.789 gm/mL.

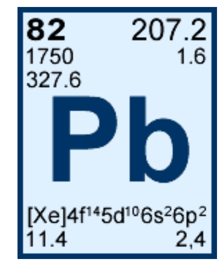
$\rho(Pb)$ = 11.34 gm/mL.

$\rho(Au)$ = 19.30 gm/mL.



Archimedes' dilemma





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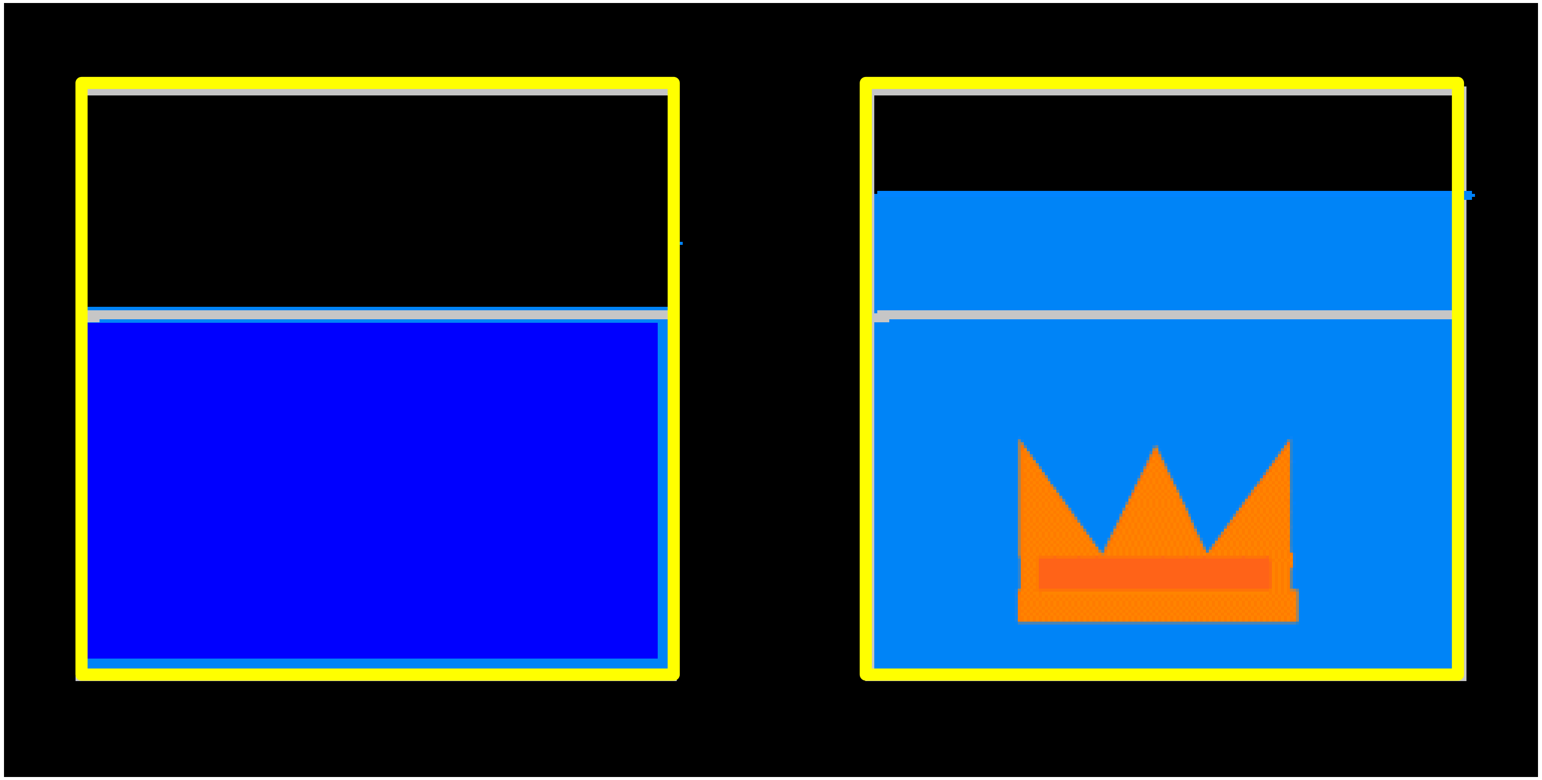
Archimedes' dilemma

It is easy to find the mass of the crown,
but how does one measure its density?

Archimedes' solution

If he seems unnecessarily excited, remember that the penalty for failing to solve the problem was death.

Archimedes' principle



The crown displaces a volume of water equal to its own volume. It is easy to figure out the volume of the displaced water. Next Archimedes realized that this explains how ships can float. The buoyancy of an object is equal the weight of the water it displaces.

Number density

We can also define a number density of a substance. This quantity is just the number of molecules per unit volume. For example,

$$\rho_N = \frac{N}{V}$$

Where N is the number of particles of molecules and V is the volume. If we want to relate the number density to the mass density we need several useful conversion factors. First, we define the number of moles, n:

$$n = \frac{N}{N_A}$$

Conversion factors

We can formally write a conversion from number of moles, n, the mass, m, using:

$$m = nM_m$$

where M_m is the molar mass. We will use this conversion factor so frequently that it is a good idea to become very familiar with it. The most common application is to determine the number of moles from the mass:

$$n = \frac{m}{M_m}$$

Concentration: molarity

The molarity is defined as the number of moles per liter. We most often consider the molarity of the solute. For example, if we dissolve 40 grams of NaCl in one liter of H_2O we have:

$$c = \frac{n}{V} = \frac{m}{M_m V} = \frac{40\ \mathrm{gm}}{\left(58.5\,\frac{\mathrm{gm}}{\mathrm{mol}}\right)(1\mathrm{L})} = 0.683\ \mathrm{M}$$

In thinking about conversions it can also be useful to understand the molarity of the solvent. What is the molarity of H_2O?

Molarity of solvent H_2O

Determining the molarity of the solvent requires a knowledge of the density. Using the density we can determine that mass of solvent in one liter. Then using the molar mass we can convert that mass to number of moles. It is particular easy for water since the density is 1 kg/L, which is 1000 grams/L.

$$c = \frac{\rho}{M_m}$$

Using the molar mass we can convert this number of grams into number of moles.

Molarity of H_2O vs. ethanol

Thus, for water we have

$$c_{H2O} = \frac{\rho}{M_m} = \frac{1000\ gm/L}{18\ gm/mol} = 55.55\frac{mol}{L}$$

Using this method we can also determine the molarity of solvent ethanol.

$$c_{Ethanol} = \frac{\rho}{M_m} = \frac{789\ gm/L}{46\ gm/mol} = 17.1\ \frac{mol}{L}$$

Concentration: molality

The molality is defined as the number of moles per kg of solvent. To convert from molarity to molality we need to use the density of the solvent.

$$\mathbf{m} = \frac{c}{\rho}$$

$$\mathbf{m} = \frac{\text{mol/liter}}{\text{kg/liter}}$$

Molarity vs. molality

We use these two sets of units for convenience in different situations. For example, in solutions of salts, acids/bases etc. we will use molarity. Molarity is by far the most common unit system in the laboratory. However, when considering the colligative properties such as freezing point depression, we will use molality. This is because the colligative property depends on the mass rather than the volume. However, for osmotic pressure we will use molarity since the osmotic pressure rise is related to volume rather than mass.

Concentration: mole fraction

The mole fraction is the number of moles of a given substance divided by the total number of moles present in the system.

$$x_i = \frac{n_i}{n_i + n_j + n_k + \cdots}$$

The convert from one of these to the other remember that the ratio of each of these quantities are equal:

$$\frac{n_i}{n_j} = \frac{x_i}{x_j} = \frac{\mathbf{m}_i}{\mathbf{m}_j}$$